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ABSTRACT

Based on research and teacher experience, this resource guide contains teaching methods and activities that teachers can use in teaching mathematics to adult basic education and General Educational Development students. The following eight sections are included: (1) an alternative method for teaching percents; (2) unit conversion or dimensional analysis; (3) reading and writing across the curriculum; (4) integrating mathematics and language arts; (5) problem solving in mathematics education (6) 26 selected references for problem solving; (7) 50 selected problems; and (8) mathematics hints and helps for adult learners. (KC)

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A TEACHER RESOURCE MANUAL FOR ABE/GED MATHEMATICS TEACHERS

FUNDED THROUGH A "353" OPI TEACHER TRAINING PROJECT
1992-1993

WRITTEN BY:

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"A METHOD TO YOUR MATHNESS"

A Teacher Training Project

Funded through a "353" Grant from the Montana Office of Public Instruction
Department of Adult Education

by

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Over the past few years, it has become apparent that there is little or no training available in Montana that is geared specifically for ABE math teachers. Many who teach ABE math are certified in other subject areas and have expressed a desire to get together with other teachers across the state to share ideas for teaching mathematics in the ABE classroom. There is also a lack of information in the existing literature about ABE mathematics teaching methods and curriculum. With these ideas in mind, the presenters wrote a grant for a "353" Teacher Training project. This "Teacher Resource Manual" has been funded by the grant, which was obtained through the Office of Adult Education, Montana Office of Public Instruction.

In writing the manual, we tried to include teaching methods and activities that we use and have found to be successful. Our ideas come from a combination of research, experience, and training in the area of mathematics education. We hope you will find the ideas to be useful in your classroom, either as they are written or with adaptations to fit your own needs.

ACKNOWLEDGMENTS

*Thanks to Christa Steiner for providing us with the title
"Method to Your Mathness"!
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AN ALTERNATIVE METHOD FOR TEACHING PERCENTS THE PERCENT " T "

by
ROSE STEINER

Teaching percentage problems has always been difficult. There are two methods most often taught. The first method solves percents by comparing ratios. For example, in determining what percent of 30 is 15, an attempt is made to find a number that compares to 100 in the same way that 15 compares to 30. This proportion method of solving percents does not help the student fully understand percents and is difficult to apply to word problems involving percents. The formula method uses the process of finding a missing factor to solve percentage problems. Using the above example, the statement is written as an algebraic equation, $? \% \times 30 = 15$, and then solved by dividing 30 into both sides of the equation and changing the answer into a percent. This algebraic equation approach seems appropriate for the higher level student who plans to continue with algebra. Before 1982, I was teaching the formula or the proportion method to solve percent problems. A student using these methods would seldom have a better than 75% accuracy. I have found that the PERCENT " T " method often improves the accuracy and retention of the student 15-20%. It is the method used presently in my classroom.

Before introducing the PERCENT " T ", it is important to remember that percents do not exist as mathematical numbers. In other words, you cannot multiply or divide by a percent. Percents do, however, aid in comprehension. For mathematical computations, though, percents must always be converted (renamed) to the appropriate decimal or fraction equivalent. The PERCENT " T " allows the student to visualize the percent problem more quickly than any other method used. But this method still requires the student to practice and memorize the six conversion rules commonly used in percents:

1. DECIMAL to FRACTION: Put the number over the place value and reduce or Read it, (w)Rite it and Reduce it--the 3R's.
2. DECIMAL to PERCENT: Move the decimal two places to the right and add a percent sign.
3. FRACTION to PERCENT: Divide the bottom into the top, move the decimal two places to the right, and add a percent sign; or
fraction $\rightarrow$ decimal $\rightarrow$ percent.
4. FRACTION to DECIMAL: Divide the bottom into the top.
5. PERCENT to FRACTION: Put the percent number over 100 and reduce.
6. PERCENT to DECIMAL: Move the decimal two places to the left and drop the % sign.

The PERCENT "T",

PART	
TOTAL	%

is a visual adaptation of the algebraic equation method of solving percents. When using this method, the percent equation,

__ % of __ is __,

is changed to the percent statement,

PERCENT of TOTAL is PART.

Using an algebraic equation, " of " changes to multiplication and " is " becomes an equal sign. Some refer to the PART as the " is " and the TOTAL as the " of ".

$$\text{PERCENT} \times \text{TOTAL} = \text{PART}$$

In the percent statement, the PART is relatively easy to determine by multiplying the PERCENT and the TOTAL. The difficulty occurs when the PART is known and the TOTAL or the PERCENT is unknown. The students often know they need to divide, but seem to guess on which number to use for the divisor. It is for this reason that the PERCENT "T" exists.

UNDERSTANDING THE PERCENT " T "

Just as numbers that are represented by letters imply multiplication when written side-by-side (2a, 4b, xy), the TOTAL and PERCENT found side-by-side in the PERCENT " T " also indicate multiplication. This agrees with the percent statement.

$$\frac{\text{TOTAL}}{\text{PERCENT}} \times \% = \text{PART}$$

The horizontal line in the " T " represents a fraction bar. Any fraction implies division (top $\div$ bottom). For example, $1/2$, $3/4$, miles/hour all imply division: $1 \div 2$, $3 \div 4$, miles $\div$ hour, respectively. The following fractions are found in the PERCENT " T ":

$$\frac{\text{PART}}{\text{TOTAL}} \quad \frac{\text{PART}}{\text{TOTAL}}$$

$$\frac{\text{PART}}{\%} \quad \frac{\text{PART}}{\text{PERCENT}}$$

These fractions also imply division. When the PART is divided by the TOTAL, the PERCENT is found. When the PART is divided by the PERCENT, the TOTAL is found.

$$\text{PERCENT} = \frac{\text{PART}}{\text{TOTAL}} \quad \text{TOTAL} = \frac{\text{PART}}{\%}$$

Simply put, if the two bottom terms are given, you multiply. If one top and one bottom term are given, you divide (top $\div$ bottom).

The PERCENT " T " in its applied form appears as follows:

$$\begin{array}{c} \text{is} \\ \text{PART} \\ \hline \frac{\div}{\text{of}} \quad \times \quad \frac{\div}{\%} \\ \text{TOTAL} \quad \text{PERCENT} \end{array}$$

WORKING PERCENTAGE PROBLEMS WITH THE PERCENT "T"

The PERCENT "T" is first applied to simple equations, where either the PART, TOTAL, or PERCENT is unknown. The two known quantities are placed appropriately in the "T". There are three basic types of percent problems: 1) to find the part, when the total and the percent are given; 2) to find the percent, when the part and total are given; 3) to find the total, when the part and the percent are given. When the PERCENT "T" is understood and the two given numbers are placed correctly in the "T", the calculation to perform becomes obvious. Therefore, depending upon what is unknown, one of the following three components of the PERCENT "T" is used:

$\begin{array}{c} ? \\ \hline \text{TOTAL} \times \% \end{array}$	$\begin{array}{c} \text{PART} \\ \div \\ \hline \text{TOTAL} \quad ? \end{array}$	$\begin{array}{c} \text{PART} \\ \div \\ \hline ? \quad \% \end{array}$
---	---	---

EXAMPLES: (Remember, percents must be changed to usable form before doing calculations.)

1. 50% of 4 is ____
$$\begin{array}{r} ? \\ 4 \overline{) 50\%} \\ \underline{20} \\ 30 \\ \underline{20} \\ 10 \end{array}$$
 $4 \times .5 = 2.0$ 50% of 4 is 2

2. 2 is ____% of 10
$$\begin{array}{r} 2 \\ 10 \overline{) ?} \end{array}$$
 $10 \overline{) 2.0} = .2 = 20\%$ 2 is 20% of 10

3. 40% of ____ is 8
$$\begin{array}{r} 8 \\ ? \overline{) 40\%} \\ \underline{40} \\ 0 \end{array}$$
 $.4 \overline{) 20} = 50$ 40% of 20 is 8

USING THE PERCENT " T " IN WORD PROBLEMS

We use percents in many different situations every day. Percents are used in business, sports, budgets, taxes, clothing labels, and scientific results. "In the middle ages, merchants commonly used percents even before the appearance of the decimal number system. After the introduction of the decimal system, the idea of percents was no longer needed. However, it was so deeply woven into business, professional, and everyday life that use of the term continues today." ("Percentage," The World Book Encyclopedia, (1988), Vol. 15, p. 280.) Percents are helpful in solving everyday problems and in showing relationships between numbers. To solve any percent problem, two of the three terms in the percent statement must be known.

The PERCENT " T " initially is used with simple equations but adapts very well to word problems. Using the proportion or formula method, it is necessary to first write the percent statement before applying either method. This additional step is not necessary when using the PERCENT " T " method. Using this method, one must only identify which two of the three terms (TOTAL, PART, PERCENT) are known and place them properly in the " T ". Since the student must identify which two of the three terms the given numbers represent, here is a review of the three terms used in percents:

- **PERCENT:** This is the easiest number to identify because it has the % symbol. Even though only one % is given, there is another that can be determined. For example, if 80% of the games played are won, we know that 20% are lost because together they make the whole or 100%. If 60% of the students have done their homework, then 40% have not. If there is a 30%-off sale, then 70% is left to pay at the sale.
- **TOTAL:** This is the number that represents all of something. It is all the games played. It is all the students in a class. It is the original price of something. Many times the total is referred to as the "of" phrase because the total is the number following the word "of". Some books call it the base. In the sentence "25% of 40 is 10," the total is 40.
- **PART:** This is the amount that is being compared to the total. It is the number of games that are won. It is the number of students doing their homework. It is the amount now after an increase or decrease has taken place. It is the "is" phrase. It is called the percentage in some books. In the sentence "25% of 40 is 10," 10 is the part.

Notice each percent and each part has a label. In order for the "T" to work, the PERCENT and the corresponding PART **MUST** have the same label. For example, if there is a PART empty, there will be a PERCENT empty. Likewise, in looking for the PERCENT increase, the PART increase must be known. Remember, opposite percents always total 100%.

EXAMPLE:

If it is known that 80% of the games are won, it is also known that 20% are lost.

PART	
TOTAL	80% WON 20% LOST

EXAMPLE:

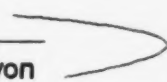
If there is a 30%-off sale, it is known that 70% is left to pay at the sale.

PART	
TOTAL	30% OFF 70% PAID

In word problems involving two corresponding or opposite percents, adapt the "T" so the two percents fit into the percent box as shown.

EXAMPLE:

Joe's team won 75% of their games. If they lost 9 games, how many games did they play?

9 lost		9 lost
? 75% won		? 75% won 25% lost

The labels do not match
so we must look for the
% lost. $100\% - 75\% = 25\%$

The student would divide 9 by 25% (.25), and get 36 for the total.

When the percent problem involves an increase or decrease, using the time frame of "original" and "now" will give the student another method of analyzing the word problem.

now is PART	
original of TOTAL	Two %'s

EXAMPLE:

Pencils sold for \$6 last year. This year the same pencils sold for \$10.50. What is the percent of increase in price?

10.50 now
6 | ? increase

The labels do not match. We want the % of increase, so we need to find the amount of increase.

$$\$10.50 - \$6.00 = \$4.50$$

4.50 increase
~~10.50 now~~
6 | ? increase

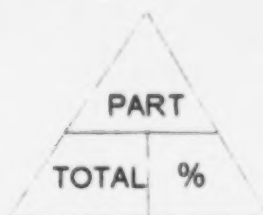
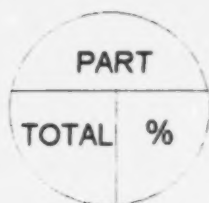
When 4.50 is divided by 6, the 75% increase is found.

As the student completes the "T", these questions need to be asked:

1. What am I looking for: PART, TOTAL, or PERCENT?
2. Which do I know: PART, TOTAL, or PERCENT?
3. What are the labels on the PART and PERCENT?
4. Do the labels match? If not, can I find another % or change the part in some way so they do match?

VARIATIONS OF THE PERCENT " T "

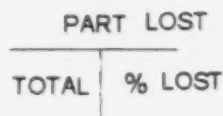
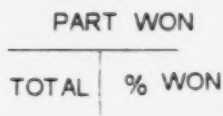
There are several variations of the Percent " T ". Two variations are only different in appearance from the " T " already presented. It is the same " T " but is formed within a circle or a triangle. These are referred to as the percent circle or percent triangle.



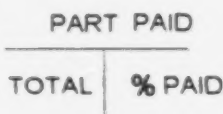
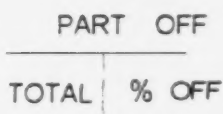
There is a whole set of " T "s used in real estate sales. See Appendix A for a list.

An alternative method uses two " T "s in word problems involving two corresponding or opposite percents, such as:

- Percent won and percent lost of games played (total):



- Percent off (discount) and percent paid (sale price) at a sale:



Once it is known that two " T "s are necessary, the percents and the parts are labeled accordingly in each " T ". Remember, in order for each " T " to work, the percent and the part in each " T " **MUST** have the same label. If the labels on the percents are identified, the same labels work for the parts. For example, if there is a percent won and percent lost, then there will be a part of the games that was won and a part of the games that was lost. After a student advances in this method, there is not always a need to draw both " T "s. The student only needs to know if they both exist.

In the first two "T"s, the TOTAL is the total games played. In the last two "T"s, the TOTAL is the original, normal, or list price. When using this method to complete the "T", the student asks slightly different questions:

1. Is there another PERCENT, and if so, what is it? Label both percents.
2. What am I looking for: PART, TOTAL, or PERCENT? If it is a part or percent, which one is it? Label the "T".
3. Which do I know: PART, TOTAL or PERCENT? If it is a part or percent, which one is it? Label the "T".

The last two questions can be asked in any order. If it is not easy to recognize the term that is unknown, the known terms usually can be readily identified. However, it is important to always begin with the first question.

EXAMPLE:

Joe's lawyer charges 30% of the amount granted by the court. If Joe has \$3500 left after paying the lawyer, how much did the court grant Joe?

First, the student decides there is another percent. If 30% goes to the lawyer, then 70% goes to Joe. Next, the student determines what is known or unknown. Knowing that the labels on the percents are also the labels on the parts, the student may ask if \$3500 is the lawyer's part, Joe's part, or the sum of the two parts (the total). If that gets the student nowhere, the student will address the remaining question. Is the amount the court grants Joe (the unknown), Joe's part, the lawyer's part, or the sum of the two parts? Following these steps helps the student determine that \$3500 is not the sum of the two parts, but rather Joe's part, and the unknown is the total. The following "T" results:

3500 JOE	
?	70 % JOE

Applying the time frame in a percent problem may seem confusing to some using this method. However, it need not be.

EXAMPLE:

Pencils sold for \$6 last year. This year the same pencils sold for \$10.50. What is the percent of increase in the price?

First the student determines if there are two percents. If the answer is not obvious, the student continues on to the next question, what am I looking for: part, total, or percent? The percent is the unknown in the problem. Therefore, "?%" can be written in the percent box of the "T". The label for the percent is determined by asking what percent is being found. Once it is determined that the percent increase is the unknown, the part and the percent areas of the "T" are both labeled as increase. The student decides what the part increase is by subtracting \$6 from \$10.50. Finally, the student determines whether the increase was on \$6 or \$10.50. These steps result in the following "T":

4.50 INCREASE	
6	?% INCREASE

FINAL COMMENTS ON THE PERCENT "T"

These are the advantages I have seen in my classroom by using the PERCENT "T" method:

1. The student becomes efficient at fraction, decimal, and percent conversions.
2. The student is aware of the components of the problem, rather than the words used, especially with word problems.
3. The student is able to visualize the "T", which easily converts to a fraction.
4. The student learns one formula instead of three.
5. It is a method different from what the students have used before. Most adult students have failed at percents previously. They favor a new method, rather than rehashing the old method(s).
6. The "T" lends itself well to other formulas involving multiplication:
(circumference, area, volume, interest, Ohm's Law, etc.).

$C = \pi D$	$A = LW$	$V = LWH$	$I = PRT$	$V = IR$
$\frac{C}{\pi D}$	$\frac{A}{L W}$	$\frac{V}{L W H}$	$\frac{I}{P R T}$	$\frac{V}{I R}$

These "T"s are helpful when determining the value for any of the terms found in the bottom of the "T", when the other terms are given.

Appendix A

Percent "T"s Used In Real Estate Sales

The basic "T" used in real estate has the part on top, percentage rate on the right and the whole on the left. The "T" still designates multiplication and division as it did earlier. Again the part and the percent MUST always have the same corresponding label.

PART	
WHOLE	% RATE

When calculating interest rate per annum (for one year), the part is the interest and the whole is the principal of the loan. The part that is the commission is calculated by multiplying the sale price (the whole) by the commission rate.

INTEREST	
PRINCIPAL	% RATE

COMMISSION	
SALE PRICE	% RATE

Taxes are calculated by multiplying the tax percentage rate and the assessed value. Depreciation is determined by multiplying the rate of depreciation by the value of the asset.

TAXES	
ASSESSED VALUE	% RATE

DEPRECIATION	
ASSET VALUE	% RATE

The seller's net profit is determined by multiplying the seller's percent by the selling price. The return on investment is the final calculation. The net operating income is determined by multiplying the rate of return by the original value of the investment.

NET PROFIT	
SELLING PRICE	% SELLERS

NET OPERATING INCOME	
INVESTMENT VALUE	% RATE

UNIT CONVERSION or DIMENSIONAL ANALYSIS

by

ROSE STEINER AND MARCIA PAPPAS

Units are names or labels on numbers. For instance, if we have 10 miles, "10" is the number and "miles" is the unit. If we have 24 hours, "24" is the number and "hours" is the unit. "Unit conversion" simply means changing a value from one unit into another, arriving at a corresponding new number. Ratios and proportions are often used to solve this type of problem. For example, if we are to change 2 feet into inches, we use the ratio of "12 inches : 1 foot" to determine that there are 24 inches in 2 feet. This is done formally, with a proportion problem, or informally, just by thinking " $12 \times 2 = 24$ ". This is where many students run into trouble. They know that 12 inches is equal to 1 foot, but they are unsure whether to multiply or divide, so they **guess**. Some students think about whether they are changing to a smaller unit (hence, they want a larger number so they know to multiply) or to a larger unit (hence, they will get less of them, so they know to divide). There is a method of doing unit conversion problems that eliminates the need to "think" or "guess" whether to multiply or divide. That method is **DIMENSIONAL ANALYSIS**.

Dimensional analysis is just another approach to converting values from one unit to another, so-called because it uses a method of "analyzing the dimensions" of the values besides working with the numbers themselves. Dimensional analysis was probably first taught in the chemistry classroom in the mid-60's. Before its use, only one chemistry conversion at a time could be completed with the ratio-proportion method. Using this new method the complete problem was set up before any of the math was computed. Sometimes the setup of the problem covered a whole sheet of paper. At first, not all chemistry professors were aware of this technique, so students using dimensional analysis were able to finish a problem before the teacher who was using the old ratio-proportion technique.

DIMENSIONAL ANALYSIS USES 3 BASIC CONCEPTS:

1. THE NUMBER ONE (1) IS THE MULTIPLICATIVE IDENTITY.

That is, you can multiply or divide any quantity by one (1), and the result will still be equal to the original quantity.

$$6 \times 1 = 6$$

2. THE NUMERAL ONE (1) CAN BE WRITTEN IN MANY DIFFERENT WAYS WHEN WRITTEN IN FRACTION FORM.

$$1 = \frac{2}{2} = \frac{3}{3} = \frac{7}{7} = \frac{41}{41} = \frac{15}{15} = \text{etc.}$$

As long as the numerator of the fraction equals the denominator of the fraction, the fraction is equal to one (1). Any definition involving units can be made into a conversion factor equal to 1. For example, since you know, by definition, that 12 inches equals 1 foot, you may say:

$$1 = \frac{12 \text{ inches}}{1 \text{ foot}} \quad \text{or} \quad 1 = \frac{1 \text{ ft}}{12 \text{ in}}$$

There are an infinite number of unit definitions (conversion factors), each equal to 1.

$$1 = \frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{2 \text{ lb}}{32 \text{ oz}} = \frac{1 \text{ meter}}{100 \text{ centimeters}} = \frac{365 \text{ days}}{1 \text{ year}} = \frac{3 \text{ ft}}{36 \text{ in}}$$

3. FRACTIONS MAY BE REDUCED OR "CANCELED" IF THE NUMERATOR AND DENOMINATOR HAVE A COMMON FACTOR.

For example, in the problem below, you cancel the common factor of four (4), and the common factor of five (5).

$$\begin{array}{ccc} 1 & 1 & \\ 4 & 5 & \\ 5 & \times & \frac{1}{16} = \frac{1}{4} \\ 1 & 4 & \end{array}$$

With dimensional analysis, not only do numbers cancel, but units that are exactly the same can also cancel. For example:

$$\frac{3 \text{ hr}}{1} \times \frac{60 \text{ min}}{1 \text{ hr}} = 180 \text{ min}$$

DIMENSIONAL ANALYSIS USES ALL OF THESE IDEAS TOGETHER:

1. DETERMINE THE UNIT YOU WANT IN THE ANSWER AND THE OTHER UNITS INVOLVED IN THE PROBLEM.
2. START WITH THE ORIGINAL UNIT.
3. MULTIPLY IT BY ONE (1) WRITTEN AS A "UNIT DEFINITION".
4. CANCEL THE LIKE UNIT NAMES.
5. REPEAT STEPS 3 AND 4 UNTIL YOU ARRIVE AT THE FINAL UNIT!

To set up a dimensional analysis problem, you must know what unit is given and what unit you are seeking in the answer. Start at the left margin of your paper with the given unit in the numerator. (When in doubt put the original unit on top.) To get rid of the original unit, place that unit in the next fraction's denominator, where it will be ready to cancel later. Search for a unit definition that will get you closer to the unit you want in the final answer. This becomes the new fraction's numerator. The resulting "unit definition" must be equivalent to 1. Continue this procedure until the unit has changed into the actual unit you are seeking in your answer. Cancel the unit names after the unit definitions have all been verified. The last step is to cancel any common factors in the numbers; then multiply and divide the remaining numbers, completing the math. This results in a new unit with its corresponding numeral.

EXAMPLES:

1. 4 feet = _____ inches

$$\frac{4 \cancel{ft}}{1} \times \frac{12 \text{ in}}{1 \cancel{ft}} = 48 \text{ in}$$

Use dimensional analysis to manipulate units even if a direct one-step definition is unknown:

2. 6 1/2 days = _____ seconds

$$\frac{13 \text{ day}}{2} \times \frac{24 \text{ hr}}{1 \text{ day}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{60 \text{ sec}}{1 \text{ min}} = 561,600 \text{ sec}$$

3. 35 2/3 mm = _____ dm

$$\frac{107 \text{ mm}}{3} \times \frac{1 \text{ m}}{1000 \text{ mm}} \times \frac{10 \text{ dm}}{1 \text{ m}} = 0.3567 \text{ dm}$$

USE DIMENSIONAL ANALYSIS WHEN SOLVING STORY PROBLEMS.

1. Randy's living room is 10 feet by 12 feet. He wants to buy carpet that costs \$12 per square yard. How much will he pay for the carpet?

This is an example of a dimensional analysis problem that begins by using a formula, namely $A = L \times W$ (the formula to find the area of a rectangle). Do not figure out the area before you do the problem. Since it involves only multiplication, calculate the area later along with the unit conversion multiplication. (This is unlike a perimeter, which must be determined before doing the problem, since it involves addition.) In the final step of the problem, use a "definition" given in the problem, namely 1 square yard = \$12.

$$\frac{10 \text{ ft} \times 12 \text{ ft}}{1} \times \frac{1 \text{ sq yd}}{9 \text{ sq ft}} \times \frac{\$12}{1 \text{ sq yd}} = \$160$$

USE DIMENSIONAL ANALYSIS TO CONVERT SQUARE OR CUBIC UNITS, EVEN IF THE DIRECT DEFINITIONS ARE UNKNOWN.

You can solve the above problem without knowing that 1 square yard is 9 square feet. Instead of canceling both feet (sq ft) in one step, cancel them in two different steps:

$$\frac{10 \text{ ft} \times 12 \text{ ft}}{1} \times \frac{1 \text{ yd}}{3 \text{ ft}} \times \frac{1 \text{ yd}}{3 \text{ ft}} \times \frac{\$12}{1 \text{ sq yd}} = \$160$$

2. 20 cubic yards = _____ cubic inches

$$\frac{20 \text{ yd yd yd}}{1} \times \frac{36 \text{ in}}{1 \text{ yd}} \times \frac{36 \text{ in}}{1 \text{ yd}} \times \frac{36 \text{ in}}{1 \text{ yd}} = 933,120 \text{ cu in}$$

3. The Smith's want to know if they can afford to pour a concrete driveway. They have a spot 54 feet long, 24 feet wide and 4 inches deep. If concrete is \$50 per cubic yard, how much will the driveway cost?

$$\frac{54 \text{ ft} \times 24 \text{ ft} \times 4 \text{ in}}{1} \times \frac{1 \text{ yd}}{3 \text{ ft}} \times \frac{1 \text{ yd}}{3 \text{ ft}} \times \frac{1 \text{ yd}}{36 \text{ in}} \times \frac{\$50}{1 \text{ cu yd}} = \$800$$

USE DIMENSIONAL ANALYSIS TO SOLVE THESE PROBLEMS:

1. If there are 52 M&M's in a 2-ounce package, how many M&M's would there be in a 1-pound bag?
2. If there are 3 yarks in 1 zorb, 5 yarks in 2 gumbles, and 7 gumbles in 4 pods, how many zorbs are there in 12 pods?
3. How many hours have you been alive? (Round your age to the nearest year first.)
4. One 25-pound bag of fertilizer covers 3,000 square feet. If my lawn is 30 yards by 50 feet, how many bags of fertilizer will I have to buy at the store to cover the entire lawn?
5. Jeff bought new carpet for his living room at \$25 per square yard. The room measures 18 feet by 20 feet. How much did the carpet cost?
6. How many cubic feet are there in a box that measures 36 inches by 60 inches by 4 feet?
7. The excavating company charges \$15 per truckload. How much would it cost to haul away the dirt from an excavation that is 25 feet by 27 feet by 8 feet, if each truckload contains 12 cubic yards?
8. How many bushels of corn can be stored in a crib measuring 15 feet by 6 feet by 10 feet, if there are 1.25 cubic feet to the bushel?
9. A water tank has a diameter of 5 yards and is 35 feet high. If there are 7.5 gallons in 1 cubic foot, how many gallons of water will the tank hold?
10. My flowerbed has a base of 10 feet, a height (altitude) of 2 yards, and a depth of 6 inches. Will one 0.5-cubic yard bag of decorative rock completely fill the bed?

READING AND WRITING ACROSS THE CURRICULUM

by
KATHIE DAVIAU

As content teachers we understand the need to transmit content, but also how to teach students how to learn content. Teaching students how to learn content will not only allow our students to learn more content, but they leave our classrooms with skills that enable them to continue learning on their own. Five principles are fundamental to almost all instruction (Santa, et al, 1988):

1. We need to incorporate practical and theoretical ideas about textbook organization,
2. We need our students to be active learners,
3. We need to teach our students to use a variety of learning strategies and to monitor their own learning,
4. Writing is essential for content learning, and
5. We need to use direct instructional methods, following a sequence of teacher demonstration, guided practice, and independent application.

We need to choose our textbooks according to both contents and overall organization. A well-written textbook will allow students to learn more quickly and easily. Organization can aid the student in comprehending the contents. See Appendix A for sample textbook evaluation forms.

A student's background knowledge is the strongest determiner of comprehension. The more a student knows about a subject, the better their comprehension will be. Therefore, as teachers, we need to know what previous knowledge the students have and whether we initially need to develop the topic. Teaching important mathematical concepts before the students attempt independent work will greatly enhance the amount of comprehension. Knowledge is the strongest determiner of comprehension. A pre-writing exercise or post-reading exercise is beneficial to the student. The key to success is "front loading" the exercise. Make sure your students have sufficient background knowledge to make the writing assignment a success. This allows the students to become more active in their learning.

We need to show students how to organize their ideas into meaningful categories, then how to add details to their categories, or add new categories to their organization. This method works well as a pre-reading or pre-writing exercise, or it works as a post-reading exercise. If we as teachers show our students a variety of methods and strategies, the student is much more likely to attempt a problem. The knowledge that there is more than one correct solution or strategy will give the student the confidence to begin the problem.

Taylor (1982) found that students who wrote an essay explaining a specific statistical technique did better on a traditional computational exam on that technique than their peers who had done the traditional computation exercises as homework. He concluded that writing caused the students be more involved in learning and

forced the students to understand the concepts more completely. Writing will help students learn content material.

Finally, we want to teach reading strategies directly. Durkin (1978-1979) noted that in 17,996 minutes of reading instruction, reading comprehension received 45 minutes of instructional time. Seat work activities tested, but did not teach, comprehension. If we allow the reader to become an active, creative, learner, the student will benefit now and in the future. The following ideas are just a few that are adaptable to fit the needs of your classroom.

Teach your students to make learning guides. The guides could be 2, 3 or 4 column notes to help with organization. Examples:

Main Idea \ Details

Picture \ Formulas \ Explanations

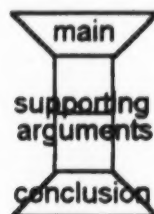
Main Idea \ Details \ Definitions \ Vocabulary

Main Idea \ Details \ Definitions, Vocabulary \ Formulas

Concepts \ Definitions \ Formulas \ Questions

Teaching the student how to organize material in this manner gives the student the background knowledge and organization necessary for further work. The student will now be able to read or write more extensively on the subject material.

Teach the students how to write with the spool method. This method uses 3 components: the main idea, the supporting arguments, and the conclusion. The main idea could be a question, a fact, a quotation, or a statement. The supporting arguments, arranged from the weakest first to the strongest last will explain why the main idea is true. There should be at least three supporting arguments. The conclusion will restate the lead-in; it is the "therefore" statement.



Have the students (and yourself) keep a content journal. The journal should be informal and non graded, concentrating on personal reactions. The journals, used to solve problems, answer questions, write problems, express opinions, summarize ideas, record daily work, brainstorm ideas about content, or clarify questions, give the students daily writing in a non-threatening format.

Teach the students to write a RAFT paper:

R stands for Role: the role that you will take for this paper.

A stands for Audience: to whom you will be communicating.

F stands for Format: what mode of expression used.

T stands for Topic: what the paper discusses.

Examples:

R: student

A: diary

F: entry

T: explain how to convert fractions to percents

R: decimal

A: students

F: story

T: tell why I move like I do

R: businessman

A: students

F: letter

T: explain why percents are so important

R: fraction

A: student newspaper

F: poem

T: explain how students misuse her

Categorizing is another method for improving comprehension. As a pre-reading exercise, start with the general topic. Brainstorm ideas associated with the topic: categorize ideas into sub-topics. Introduce new terminology, vocabulary, or concepts at this point. Let the students categorize these ideas into the sub-topics. Then write the summary. As a post-reading exercise, start categorizing with the details and vocabulary. Then organize the categories into main ideas.

Have the students write story problems. Post some on a bulletin board, especially some with too little or too much information. Have other students solve the problems, or rewrite the problem in their own words.

Have the students write the rules or algorithms. There is no better way to organize the students' thinking than to encourage them to write the rules.

Vocabulary is a critical aspect of all content areas. Some examples of how to incorporate vocabulary into a writing exercise are:

1. Vocabulary journals: Have the student write the word, a synonym, a definition, a sentence containing the word, and a picture (if applicable)
2. Conversational: Listening, talking, reading, and writing are four different modes to reinforce the new vocabulary. Discuss the words as a whole class first (listening). Then have the students break into groups to talk about the vocabulary (talking). Afterwards, have the students read an article or paragraph that includes the new vocabulary (reading). As the fourth component, have the students write a summary on the vocabulary words (writing).
3. Sentence Creation: Have the students list the vocabulary words and then review them in a discussion. As a teacher, write some example sentences for the students. Then have the students write their own sentences, either as groups or individually.

Micro themes are an excellent method to have the students write short essays on a 5x7 card. Give the same assignment to everyone to encourage group work. Sample topics would be summaries, rules, support a position, data analysis, problem solving, alternative methods to a problem, test questions, point of view, interview questions for a mathematician, hypothetical situations, paraphrase a topic to explain it to someone else, radio advertisement to sell a mathematical process, make up a math word and define it, make up math slogans for bumper stickers, etc. See Appendix B for a sample writing exercise.

A good assignment is short and sweet. Let the students use writing as a tool to aid learning (Abel and Abel, 1988). Writing improves with many writing activities (Knoblauch and Brannon, 1983). Many short assignments, rather than one or two major papers, will develop a proficiency in writing. The components of a good writing assignment are:

1. a clear statement of the task,
2. a specific purpose of the assignment,
3. a description of the final product,
4. steps and time lines that will be followed,
5. evaluation criteria that will be used.

As mathematics teachers, we do not need to correct grammar. It is best to respond to the paper on a whole. Evaluate the students on organization, details, and clarity instead of spelling or grammar. The purpose of the assignment is to help the student learn content. Too much stress on spelling or grammar may defeat this purpose (Maimon, 1988).

In conclusion, writing is a way of thinking. Thinking should be an integral part of the mathematics curriculum. If we teach our students how to organize their ideas through writing, actively read for understanding, and write to learn content, our students will become more independent, productive members of society.

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Appendix A

Mathematics Content Area Text Assessment

Name of Text _____
Author(s) _____
Copyright _____
Publisher _____

A. Overall Structure of Book

1. Table of Contents
2. Glossary of mathematical terms, symbols, formulas
3. Index
4. Appendix

B. Overall Content

1. Does the content of the text reflect what you feel are essential concepts in your courses?
2. Examine the topics presented in each chapter. Does the content flow in a logical progression from simple to more complex?
3. Does the text reflect the logical progression of mathematical concepts which you feel are appropriate?

C. Organization of Chapters

1. Do the topics and sub-topics specify the main ideas of the chapter?
2. Does the author begin a new unit or chapter with a general introduction of the chapter content?
3. At the end of a selection or chapter, does the author review essential concepts?
4. Are review sections, questions, and problems keyed to specific book sections so that students can readily review concepts they did not originally understand?
5. Does the author highlight (underline, bold print, italics) key concepts and vocabulary within the text?

D. Content within Chapters

1. Does the author provide clear definitions of unfamiliar vocabulary and new concepts?
2. Does the author introduce a new concept with enough good examples so that the concept is comprehensible?
3. In introducing a new concept, does the author use motivational devices (practical examples, cartoons, etc.) to make the reading interesting?
4. When appropriate, does the author provide practical applications of mathematical concepts?
5. Are the mathematical exercises sufficient?
6. Do exercises progress from simpler to more difficult problems?

E. Sentence Level

1. Are sentences consistently short and clear?
2. Are most verbs in the active voice?
3. Does the author use explicit signals to indicate sequencing of ideas (*first, second, third*)?
4. Does the author use emphasis words to indicate important concepts (*most of all, a key feature, a significant factor*)?
5. Does the author use explicit signals to indicate comparisons (*but, however, on the other hand*)?
6. Does the author use explicit signals for illustration (*for example, such as*)?
7. Does the author use explicit signals for conclusions (*therefore, as a result*)?

F. Concept Development

1. Are new concepts linked to a student's prior knowledge?
2. Are concepts first defined, then followed by clear examples?
3. Are concepts explained clearly with sufficient elaboration?

G. Vocabulary Density

1. For the evaluation of vocabulary density, count out two 100-word samples. For each selection, circle all vocabulary that might be unfamiliar to some or most of your students. Then count the number of circled words and record below. Remember, mathematical notations and formulas are classified as vocabulary.

Selection 1 _____ Selection 2 _____

2. If the total for either selection is five or more words, the vocabulary demands will very likely create some problems for your students.

Appendix B

I use this idea sheet in my classroom.

1. You are an investigative reporter. You need to interview this sequence:
1, 4, 9, 16, 25, ...
Make a list of at least four questions you will ask to discover the next number in the sequence.
2. Write a short poem or limerick which includes the rules of addition, subtraction, multiplication, and division of fractions, decimals, or algebra.
3. Write a letter home to parents or children to explain the conversion rules of percents and why they are so important.
4. Write an article for the local newspaper to explain a process in math (the percent T, conversion of measurements, long division, etc.)
5. Write a paragraph entitled "Who Am I?" Give clues that will lead to the identity of a certain number.
6. Write a recipe that explains how to build a polynomial.
7. Make a list of the important skills and rules in the workbook you are working in now. Include ideas that were new to you, plus ideas you already knew.
8. Pretend you are a decimal. Write a paragraph explaining why you work the way you do.
9. Make up a word and write a mathematical definition for it.
10. Choose a mathematical process and write a radio advertisement for it. Tell who would need it and why.
11. Write 4 story problems from the booklet you are working in now. Write one problem each from addition, subtraction, multiplication, and division.
12. The story problem from hell...
13. Find a famous mathematician in the math library and write a short summary of what he/she did.
14. Write a diary entry explaining how math class made you feel on a certain day.
15. Finish this sentence: Learning math as an adult is important because...

This is, of course, not an all-inclusive list. It merely gives each student an opportunity to choose a topic she or he will feel comfortable with.

INTEGRATING MATH AND LANGUAGE ARTS

by

MARCIA PAPPAS

As a math teacher, I have always felt that math cannot and should not be taught in a vacuum. What good is math if it doesn't apply to anything else in the real world? And what good is math if we can't use words to talk about it, to explain it, to make it work? And finally, what good is math if we can't use it to explain other things? I've often wished that I could have an English teacher at my side so that I would feel more comfortable talking about how math DOES relate to "real life", how math and writing, math and reading, etc., are related. I have had my wish granted this year, as I have been fortunate enough to "team-teach" with one of my colleagues, Norene Peterson, a language arts teacher.

Integrating math and language arts is not a new idea to those of you who teach both of these subjects, especially in a GED prep class. But too often, even if we are teaching more than one subject, we tend to compartmentalize each of them--math is from 9:00 to 10:00 and language arts is from 10:00 to 11:00 and never the twain shall meet! But that's not the way it works in the real world. We all know that we use both of these subjects every day, but students sometimes don't realize that they are even related. How often have you heard a student say in a math class, "I can't do these problems because they have words in them and that's English class, not math!" With a little ingenuity and some planning ahead of time, I believe we can teach some relevant lessons that include both subject areas and show our students the value and the reality of both math and English. I want to share some of the projects that Norene and I have done with our combined language arts/math class, in the hopes that you can use some of these ideas, adapt them to your own situation, and think of new ideas that you will pass along to us and to other teachers.

ACTIVITY #1: SPREADSHEETS, GRAPHS, AND WRITING

DESIRED OUTCOMES:

Students will:

- ▷ Use a spreadsheet and charting program
- ▷ Interpret a graph or chart
- ▷ Write about their interpretation

MATERIALS:

- ▷ Computers
- ▷ Software (MS Works, Claris Works, Apple Works, or any spreadsheet program and word processing program)
- ▷ Sample spreadsheet that you have previously created

METHODS:

1. Show the students a sample spreadsheet that you have already created. Talk about rows, columns, cells, moving the cursor, typing headings, and using formulas (which, in itself, can be a full class period discussion). Show how the spreadsheet automatically re-calculates figures if the numbers are changed.
2. Show what types of charts or graphs the spreadsheet will make. You can talk about the different types of charts and what data are most appropriate for what type of chart. You can also talk about all the terminology associated with graphs, such as vertical and horizontal axes, grid lines, units, increments, bar charts, pie charts, etc., and how to analyze and interpret a graph.
3. Do some brainstorming on possible uses of a spreadsheet. Some ideas are household budgets, test scores, attendance records, sales records, and keeping track of car expenses. The students will probably come up with many ideas.
4. Have the class choose one of the ideas and "walk them through" the creation of a small spreadsheet with made-up data to learn the steps. It is good to point out that just as they do before they write an essay, they should do some planning on paper before keying the information. We used the board as our "scratch paper" and planned the look of the spreadsheet to show the students how to do this. They should also create a chart from this spreadsheet. When it comes time to put a title on the chart, remind them how they determine a title for a paragraph or an essay that they have written. In other words, what is the main idea--what is the chart trying to show the reader?

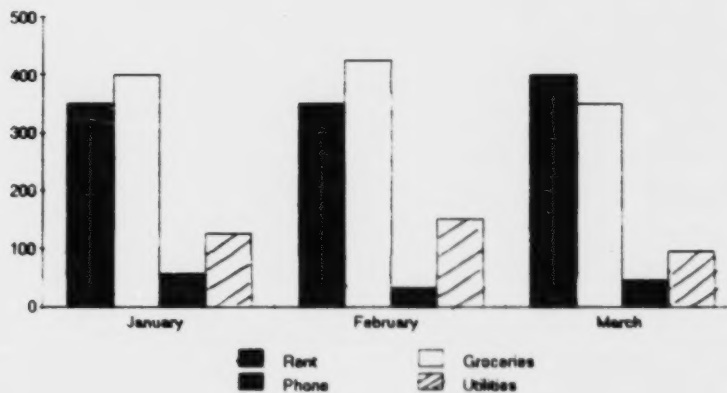
5. After everyone has done this whole-class exercise, ask each student to create his or her own spreadsheet and chart based on a certain topic, such as a personal budget for the last 3 months. Again, urge the importance of planning (just as we pre-write in English class) before keying information.
6. When they have printed the spreadsheet and chart, ask them to analyze the information and think about the questions, "Does my personal budget need to be revised? Am I spending too much money in a certain area? Do I need to increase spending for any area? How should I change my spending habits, or are they OK? Why or why not?" This is a great topic for an essay! And if you are using one of the "works" types of programs, students can easily switch to the word processing part of the program to start brainstorming ideas for the essay. They can switch back and forth on the screen between the spreadsheet and the essay (super-easy if you have Macs, or IBM's with Windows)! If you are not using an integrated program, they can refer to the hard copy of the spreadsheet and chart while they are brainstorming for the essay with the word processor.

This activity combines mathematics, computer use, and written language in a cohesive way. It also presents the students with a situation that they may see is very true to life and one that may have personal implications for them.

SAMPLE SPREADSHEET AND BAR CHART:

FIRST QUARTER BUDGET

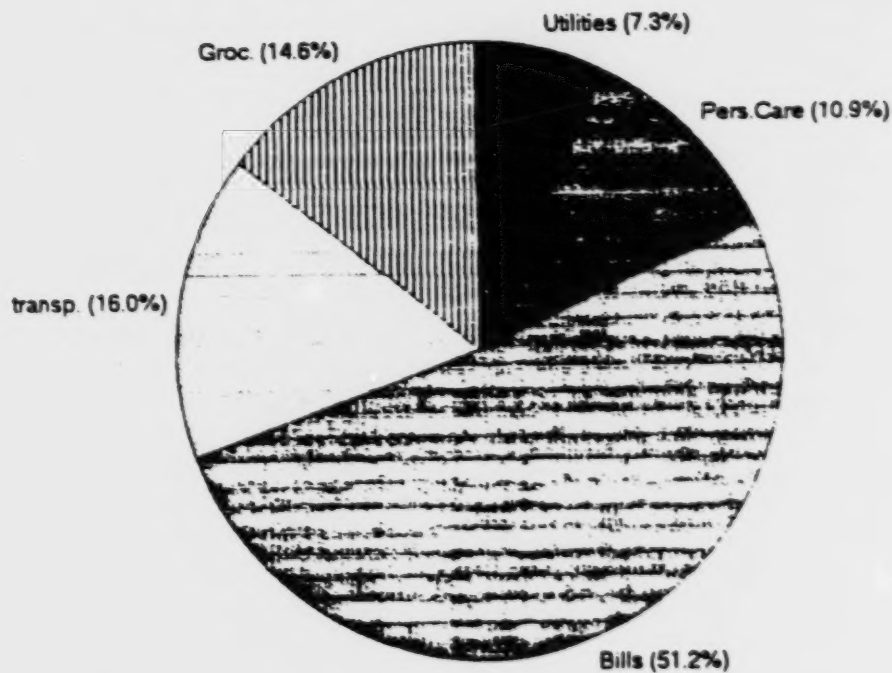
	January	February	March	Total
Rent	350	350	400	1100
Groceries	400	425	350	1175
Phone	56	32	45	133
Utilities	125	150	95	370
Totals	931	957	890	2778



This is an actual sample of a student's pie chart and budget paragraph:

BUDGET

I need to change my budget because of overexpenditures on my credit cards. I use credit cards often. They're easy, and I don't have to pay them off right away. It causes a problem in my budget because of the high interest rates which run around 18%. I should learn to use my credit cards sparingly, especially when buying large items and paying them off every month.



ACTIVITY #2: WRITING STORY PROBLEMS

DESIRED OUTCOMES:

Students will:

- ▷ Read an article from a newspaper or magazine
- ▷ Determine mathematical questions that can be generated from the article
- ▷ Identify specific information that is needed to solve each problem
- ▷ Write several "story" problems, including the question to be answered and the information from the article that is necessary to solve the problem

MATERIALS:

- ▷ Magazine or newspaper article--enough copies for each student
- ▷ Computers and word processing software are nice, but not necessary

METHODS:

1. Distribute copies of the magazine or newspaper article and let students read it.
2. Either as a class or individually, ask the students to make a list of numerical facts that are stated in the article. These facts will be used in preparation of the "story problems". You may want to list the facts on the board or on an overhead.
3. Using this list of facts, have the students write story problems that are based on this information (we have students work in pairs). You may wish to give them a certain time limit in which to write the problems.
4. Ask them to solve and keep a copy of their problems and the solutions.
5. Compile these story problems into a single list. This will be very easy if students have used a word processing program.
6. Print the entire group of problems and hand them back to the students to solve.
7. They may solve the problems individually or in groups. They will find when they attempt to solve the problems that there are grammatical mistakes, too little information given, a question that does not apply to the information, questions that simply do not make sense, etc. Ask them to make necessary corrections in the wording, spelling, grammar, etc.
8. Discuss and solve each problem and talk about both the language arts and the mathematical aspects.

ACTIVITY #3: INTERPRETING A GRAPH

DESIRED OUTCOMES:

Students will:

- ▷ Analyze a graph that is presenting information in a distorted way
- ▷ Use mathematics to explain why the graph does or does not show what it claims to show
- ▷ Discuss the implications of the way the graphing was done
- ▷ Write a paragraph or essay giving their opinion of the controversy

MATERIALS:

- ▷ Graph that is showing information in a distorted way (one that we used came from a newspaper editorial, but you could make your own graph)

METHODS:

1. Show the graph on the overhead projector and/or as individual student handouts.
2. Talk about what the graph is supposedly showing and why it does or does not achieve its goal.
3. Have students decide how the graph should be changed in order to show the true picture.
4. Discuss the reasons why the graph may have been distorted on purpose--who would benefit and how?
5. Discuss the controversy itself and talk about the pros and cons of each position.
6. Have students write a paragraph or essay giving their own position regarding the controversy, and supporting their position with facts and details.

This activity points out how easily statistics can be distorted to show a conclusion that is not true. If the topic is of general interest and is taken from current events, this can be a very potent lesson.

ACTIVITY #4: SOLVING STORY PROBLEMS

DESIRED OUTCOMES:

Students will:

- ▷ Learn a 5-step method of solving problems
- ▷ See how this is directly related to a 5-step method of writing
- ▷ Identify strategies that may be useful in solving problems

MATERIALS:

- ▷ Story problems

METHODS:

1. Remind students of the 5-step method for writing:
 - ✓ Pre-write
 - ✓ Organize
 - ✓ Write
 - ✓ Evaluate
 - ✓ Revise
2. Talk about a 5-step method for solving story problems:
 - ✓ Prepare to do the problem by reading it and re-reading if necessary
 - ✓ Organize the information and analyze all aspects of the problem
 - ✓ Work out a strategy
 - * Make a simpler problem
 - * Make a table
 - * Guess and check
 - * Draw a picture
 - * Make an equation
 - * Find key words
 - ✓ Execute the strategy
 - ✓ Review the answer and re-work the problem if necessary
3. Model the 5-step problem solving method by doing several story problems, using the blackboard, overhead, or PC viewer to show the class.
4. Hand out a page of story problems to the class and have students discuss how to apply the 5-step method to some of the problems. GED practice problems work very well.
5. Have the students do the remaining story problems either individually or with a partner. Ask them to show each step. Afterwards, you can discuss the many different methods that each person or group used in solving the problems.

ACTIVITY #5: MATH ACTIVITY & CORRESPONDING LANGUAGE ARTS LESSON

DESIRED OUTCOMES:

Students will:

- ▷ Be introduced to perimeters, areas, and volumes
- ▷ Use the associated vocabulary in a "subject-verb agreement" exercise

MATERIALS:

- ▷ Lesson plan for introducing (or reviewing) perimeters, areas, volumes
- ▷ Worksheet for students to practice using the formulas
- ▷ Worksheet using the associated vocabulary in "subject-verb agreement" problems

METHODS:

1. Present lesson on perimeters, areas, and volumes. This can be as introductory or as advanced as your students need.
2. Let students practice the concepts and how to use the formulas.
3. Present lesson on subject-verb agreement.
4. Give students a worksheet using the math vocabulary in sentences in which they must choose the correct verb, and then tell whether the sentence is mathematically correct.

MATH/LANGUAGE ARTS WORKSHEET

Underline the subject of each sentence and circle the verb that makes the sentence GRAMMATICALLY correct. Then mark each sentence either true or false from a MATHEMATICAL point of view.

1. The perimeter (is, are) the product of a shape's sides.
2. The length and the width of a rectangle (is, are) necessary to know in order to determine the perimeter.
3. All sides of a triangle (is, are) equal.
4. Each of a square's sides (is, are) equal to one another.
5. Either the circle or the rectangles (has, have) a perimeter.
6. Neither the sides of a rectangle nor the diameter of a circle (is, are) needed to find the perimeter or circumference.
7. Parallel lines (is, are) not like railroad tracks.
8. The areas of two squares (is, are) equal to their lengths times their widths.
9. The area of a triangle (is, are) equal to the altitude times the base times two.
10. π (is, are) equal to 3.41.
11. Both of the math and the language arts classes (is, are) truly exciting.
12. Neither math nor language arts (is, are) truly exciting.
13. There (is, are) three sides to every triangle.
14. Each diameter (is, are) made up of two radii.
15. Length, width, and height (is, are) needed to figure out the volume.

BONUS QUESTIONS

Two teachers who may appear to be real squares going around in circles (is, are) certainly not dense because they truly (know, knows) their areas and (has, have) depth to their thinking--contrary to their students' beliefs!

$\square$ (is, are) squared.

$\square$ (is, are) round.

PROBLEM SOLVING IN MATHEMATICS EDUCATION

by
KATHIE DAVIAU

Problem solving has many different meanings and occurs in many different disciplines and professions. Problem solving is creating new ideas, inventing new products, or troubleshooting. It is a method of inquiry and application. Problem solving is the process by which students experience the power and usefulness of mathematics in the world about them. It is mathematics in the making (House by Krulik, 1980). Problem solving is an all-encompassing term that can mean different things to different people at the same time, or different things to the same person at different times (Branca by Krulik, 1980).

Problem solving in mathematics is more specific, yet still open to different interpretations. Problem solving in mathematics has many names: solving simple word problems, solving nontraditional problems or puzzles, applying math to real-life problems, or creating and testing mathematical conjectures.

George Polya (1980) has stated that solving problems is the specific achievement of intelligence, and intelligence is the specific gift of man. If education fails to contribute to the development of intelligence, it is obviously incomplete. Intelligence is essentially the ability to solve problems. Polya further states that the first duty of a teacher of mathematics is to develop his students' ability to solve problems.

However, problem solving is not the kind of process that does or does not happen. It occurs in different ways with different people (Fellentz, 1988). Success in problem solving depends on each learner's individual differences: amount of previous knowledge, concept distinction, flexibility in formulating hypotheses, retention of the solution model, and the ability to abstract from a specific example (Kleinmuntz, 1966). The goal as a mathematics educator; therefore, should be to develop these individual differences to their optimal levels.

Ideas develop; they do not go from "not known" to "understood" in one step. Likewise, problem solving skills develop over a long training period. Without specific training in problem solving, the development of problem solving skills is slow and inefficient at best. Teaching problem solving as a specific goal must begin as early in the curriculum as possible (Tuma, 1980). Nicholas Branca (1980) has stated that problem solving must be taught specifically as a goal, as a process, and as a basic skill.

To teach problem solving as an objective, it must be independent of specific problems, of procedures or methods, and of mathematical content. With problem solving as a goal, the primary consideration is to learn how to solve problems. When problem solving is a process, the methods, procedures, strategies, and heuristics that students use to solve problems are the primary considerations. When teaching problem solving as a basic skill, the focus is on the essentials of problem solving that all students must learn (Branca by Krulik, 1980).

Branca further states that considering problem solving as a goal can influence every aspect of what a classroom teacher will do to teach math. If problem solving is reflected in the curriculum, textbooks, supplementary materials, and daily lessons, the educator has another purpose for teaching math. With problem solving as a process, an educator will now examine the relationship between skills and concepts and what role they play in the solution of various problems. Considering problem solving as a basic skill, an educator will be able to organize the specifics of the daily teaching of skills, concepts, and problem solving. Developing students' problem solving abilities is one of the most important, yet most difficult, instructional goals to achieve. An educator teaching problem solving must consider three things: curriculum, teaching methods, and teaching actions.

To establish problem solving as an integral part of the curriculum, regardless of the time involved, the educator must commit himself (Charles, Mason, & White, 1982). The classroom teacher needs to provide daily problem solving experiences for all students. Problem solving, in some form, is for everyone. Every student deserves the pleasures and benefits of problem solving, regardless of his or her ability level (Kantowski by Krulik, 1980). Daily problem solving exercises will develop positive attitudes and strengthen problem solving skills. Both complex transaction problems (problems that require two or more steps to arrive at the solution) and process problems (problems that concentrate on the process rather than the solution) need development in the classroom. The teacher should use activities that develop problem solving skills, such as:

- 1) given a problem, determine if there is enough information, extra information, or the correct amount of information,
- 2) given a problem and an answer, determine if the answer is reasonable,
- 3) given a situation without a question, make up a question, and
- 4) given a problem, determine the conditions in the problem that affect the solution.

The teacher should also pose the problems differently for high and low achievers. For example, "What is the greatest number of pieces of pizza that can be cut using 6 straight cuts?", versus "What is the greatest number of pieces of pizza that can be cut using 1 straight cut? 2 cuts? 3 cuts? 6 cuts?" The second example suggests a pattern, which would influence the choice of strategies the students would choose to use (Charles et al, 1982). Howard Johnston and Glenn Markle (1981) have outlined the following recommendations for teachers:

- 1) embed teaching and learning experiences in a problem solving format,
- 2) think hints or suggestions rather than absolute procedures,
- 3) experiment by giving less help,
- 4) don't be misled by the benefits of "teach 'n tell." Long range problem solving benefits may be higher,

- 5) use different problems with different approaches,
- 6) the process becomes the solution, not necessarily the answer. The learner must understand this.
- 7) don't protect the learner from error. The learner must be able to detect and demonstrate what's wrong and why.

Studies have shown that the teaching methods used in the classroom greatly influence the actions of the students. Before the students will have a positive attitude, the teacher must display a positive attitude toward problem solving. The teacher must place the emphasis on the process, not the product. Continually recognizing the following behaviors will develop both problem solving-skills and positive attitudes among the students: the willingness to attempt the problem; perseverance; and selection of a strategy, regardless of the elegance, or even usefulness, of that strategy (Charles et al, 1982).

The teaching actions of the educator will also greatly influence the students' ability to experience insightful, lasting, and transferable problem solving skills. Problem solving is taught and learned in five not necessarily distinct methods:

- 1) osmosis -- immerse the student in an environment of problems
- 2) memorization -- program the students to memorize the algorithm
- 3) imitation -- compare solutions so that the "expert's" solution can be shared with the group
- 4) cooperation -- use groups
- 5) reflection -- learn by doing and by thinking about what they are doing. This follows the theories of Dewey, Montessori, and Piaget (Kilpatrick by Silver, 1985).

Of course, the reflection method is the ultimate goal of the classroom teacher. Randall Charles, Robert Mason, and Catherine White (1982) have given the educator a list of ten teaching actions that will help the students reach the reflection method. Grouped according to time, the following shows the teaching actions to guide the students: before the solving begins, during the solving, and after the solving. See Appendix B for a sample problem solving activity. Before the students start working, the teacher should:

- 1) read the problem to the class, or have a student read the problem, discussing words or phrases the students may not understand,
- 2) use a whole-class discussion concerning understanding the problem, and
- 3) use a whole-class discussion with low achievers concerning possible strategies.

During the time the students are working, the teacher should:

- 4) observe and question the students to identify their progress in the problem solving process,
- 5) when the students reach an impasse, help them with hints and questions, being more directive with low achievers,
- 6) require students that obtain a solution to verify their solution, and
- 7) for students that finish early, give an extension of the problem or have students make up an extension to the problem.

After the problem solving session, the teacher should:

- 8) show the solution(s) on the chalkboard or overhead projector, discussing the students' solutions with a problem solving strategy guideline sheet,
- 9) if possible, relate this problem to previous problems, or discuss the extensions to the problem, and
- 10) if appropriate, discuss special features of the problem.

The students should also have visual access to a guideline of problem solving strategies. See Appendix A for a sample guideline. When the students reach an impasse, this gives them a visual aid with which to regroup their thoughts. The role of the teacher in the problem solving process is vital to the success of the students. The teacher provides guidance, develops general problem solving skills and knowledge, and reinforces positive problem solving behaviors.

The teacher also supervises the selection of the problems (Hudgins, 1966). The characteristics of a good problem are:

- 1) the solution to the problem involves a distinct mathematical concept or skill,
- 2) the problem can be generalized or extended to a variety of situations, and
- 3) the problem lends itself to a variety of solutions (Krulik & Rudnick, 1980).

Since the National Council of Teachers of Mathematics has stated that problem solving should be the emphasis in the mathematics classroom (NCTM, 1980), a wealth of books and materials are available from publishers and libraries to provide an educator with ample problems for the classroom. Dan Dolan and Jim Williamson (1983) not only have an extensive selection of problems in their book, **Teaching Problem Solving Strategies**, but also their philosophy is slightly different from what other publications use. Their approach is to teach problem solving just like an algorithm. The strategy to be taught must be susceptible to analysis and broken into constituent parts. These parts must be taught individually and sequentially, resulting in the development of the complete strategy.

When the students perceive enjoyable learning conditions, effective learning takes place (Gallagher by Krulik, 1980). Therefore, the teacher needs to make problem solving an experience that is not unpleasant. The educator's positive attitude, of course, is the best way to help the students enjoy problem solving. The use of the computer with problem solving software, writing computer programs, recreational mathematics, and textbook supplementation are all effective techniques to make problem solving more pleasant, appealing, and, thus, a transferable skill.

Students will attack most enthusiastically problems they find interesting and appealing. They are also more successful with interesting problems (Barnett, Sowder, & Vos by Krulik, 1980). Asking the students about themes for the problems assures the teacher that the problems are interesting to the students. Did the class just have a field trip? Would someone like to buy a car? Was there a television special? Did it snow last night? If it did snow last night, the teacher might ask, "What is the volume of the snow on the school grounds? If we could make one big ball of snow from the snow on the school grounds, how big of a ball would it be? When the ball melts, how much water will we have? How does this snow compare to record snows?" (Sowder et al, 1980.)

Educators assume that what they are teaching today will facilitate learning later (Hudgins, 1966). In other words, an educator teaches with faith in the transference of skills and knowledge. However, studies have shown that the human capability for transferring knowledge and skills from specific situations to analogical, but not identical, situations is very limited (Simon by Tuma, 1980). Two conditions that make transference possible are:

- 1) the problems must use essentially the same processes or knowledge, and
- 2) the learner needs awareness of these skills, abstracted from the specific context (Mertz, 1981)

These two conditions imply that educators need to be much more concerned in planning courses and curricula. Since the transference of knowledge is not likely to occur without these two conditions, an educator would need to predict the exact kinds of problems students will have to deal with, and ensure the coverage of those topics in the course material. However, accurate prediction is impossible. Thus, it becomes the responsibility of the classroom teacher to provide the opportunity to transfer general knowledge and problem solving skills.

Most of the difficulties students have in mathematical problem solving do not stem from the vocabulary and language of the problem (as important as it is). Successful problem solving depends upon the possession of a large store of organized knowledge in a given domain, techniques for representing and transforming the problem, and metacognitive processes to monitor and guide performance (Kilpatrick by Silver, 1985).

J. Michael Shaughnessy (1985) has outlined general problem solving "derailers":

- 1) lack of appropriate knowledge organization,
- 2) algorithmic bugs,
- 3) lack of problem solving strategies,
- 4) relinquished executive control (not monitoring own progress),
- 5) belief system (believe they cannot solve problems),
- 6) folklore paradigms (a belief that has gone unchecked or unchallenged to a point that a person is unaware that a problem exists),
- 7) inadequate problem representations and/or ill-chosen schema, and
- 8) reliance on intuitive heuristics.

Frank Lester (1985) has also listed problem solving blocks based on George Polya's four phases of problem solving:

- 1) Understanding -- expert problem solvers spend considerable time developing representations before taking specific action. Novice problem-solvers tend to be impulsive, seek closure immediately, and base their problem representations on syntactic information and contextual detail
- 2) Planning and carrying out the plan -- the major obstacle to success is recognizing an appropriate strategy
- 3) Looking back -- transference takes place only if the learner recognizes the relation of the problem to a general principle. Therefore, give direct attention to the learners by having them ask, "What have I learned? What are the key features to the solution? What attempts were made? What were my reasons?"

Douglas McLeod (1985) has stated that affective issues play a major role in a student's problem solving success. He has listed issues such as human information processing, emotions, belief systems, consciousness, tension, relaxation, causal attribution, independence, personality, confidence, anxiety, curiosity, and persistence that may help or hinder problem-solvers. The difficulties that a learner may have could possibly stem from the educational system itself. Current textbooks present material in such a manner that the students learn by rote. Thus, math students are being taught rigidity in thinking, not flexibility. Educators are teaching procedures, but not when and under what conditions these procedures will work. We show students what to do, but not why to do it. There seems to be four types of knowledge necessary to be successful at problem solving: linguistic and factual, schematic, algorithmic, and strategic (Lester by Silver, 1985). To deprive students of schematic and strategic knowledge is to deprive the students of thinking skills. These thinking skills should be the main objective of the mathematics teacher.

Empirical evidence verifies that problem solving skills are taught effectively. The question of transference of skills is still a genuine question of concern. The ability to teach problem solving skills more effectively occurs as educators continue to learn more about the general nature of problem solving processes (Simon by Tuma, 1980).

In summary, problem solving should be the top priority in all mathematics classrooms. It needs to be continuous and an integral part of every lesson, whether the material is routine or non-routine. The classroom teacher needs to make problem solving a pleasant, successful experience. The transfer of skills and knowledge must be the emphasis. Therefore, the learner must be aware of the learned skills as applied to a general principle.

Quality is at the heart of excellence: the issue is not how much is learned, but what is learned and how it is learned.

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Appendix A

This is a sample guideline for a visual aid for problem solving strategies. Each student should have this or a similar guideline during the problem solving session.

Problem Solving Strategies

Understanding the Problem

- ♥ Read the problem again.
- ♥ Write what you know.
- ♥ Look for key phrases.
- ♥ Find the important information.
- ♥ Tell it in your own words.
- ♥ Tell what you are trying to find.

Solving the Problem

Try this:

- | | |
|--------------------------|---------------------------|
| ♥ Look for a pattern. | ♥ Draw a picture. |
| ♥ Guess and check. | ♥ Make an organized list. |
| ♥ Write an equation. | ♥ Use a table. |
| ♥ Use logical reasoning. | ♥ Use objects. |
| ♥ Simplify the problem. | ♥ Work backwards. |
| ♥ Brainstorm. | |

Answering the Problem

Have you:

- ♥ Used all the important information?
- ♥ Checked your work?
- ♥ Decided if the answer makes sense?

Appendix B

A sample problem solving activity:

I have one-hour class periods. One day each week, I use thirty minutes for a problem solving activity. I make sure every student has a problem solving strategy guideline (see Appendix A) and a copy of the problem. Then I group the students into pairs (or threes, if I have an odd number of students that day).

I allow the students to read the problem by themselves. Then I read the question and clarify any questions the students ask. Then the students will brainstorm in their group. I wander from group to group to discover which strategies the groups have chosen. If a group is having difficulty getting started, I will give some hints. After five or ten minutes, I talk to the class as a whole. We talk about which strategies are being used and why. Then we talk about which strategies might work more efficiently. It is at this point I often go to the board to organize the strategies. Then the students work in their groups again. If a group finishes the problem, I make sure they have checked their answer. I have a back-up problem that is an extension of the first problem that I give to the group.

By this time, the students have either reached an impasse or have solved the problem. So, I go to the board to begin the actual solution. I do not attempt to solve the problem; I get the different groups to give me their strategies and solutions. If I have more than one strategy used, I will use all the different solutions on the board.

After the discussion of the solutions, we talk about all the strategies used and any extensions to the problem.

Let's look at a specific problem:

If you have a block of twelve stamps, three stamps high by four stamps wide, how many different combinations of four stamps can you find? Assume that the stamps may not be connected at the corners.

My first task is to group the students into pairs. I attempt to group experienced problem solvers together only because they often get the problem quickly. The experienced groups are also more capable of solving the extension problems. As for the novice and average skilled problem solvers, I do not group them according to skill level. I do try to give students different partners each week for variety's sake. Each student has a problem solving strategy guideline and the problem. I give each student about two minutes to read and reread the problem. Then, I read the problem to them and clarify any terminology. The question "What does it mean that the stamps can't be connected at the corner?" is asked. I draw a picture on the chalkboard of an example, plus I have a block of stamps to show them. I also make sure that the students realize that they have just finished the 'understanding the problem' portion of the strategy guideline.

After the class discussion of the problem, I allow the students to work on the problem in their groups. I go from group to group mostly observing. I am watching for different strategies and listening for different understandings. If a group has reached an impasse, I will give a vague hint on how to approach the problem. One group started listing all the different combinations. They found 12 different combinations and wanted me to check their answer. I suggested that they missed the shape of the "L." That hint was enough to get them back on track. One group was not doing all the flips of the "L," so I suggested that they look at some different flips.

Most of the groups are still trying to list all the combinations. I begin a discussion. I ask each group to tell what strategies or approaches they used. This allows me to analyze the different strategies that each group used. I make the comment that I sure would not like to list all the combinations. If I don't want to list all the combinations, what are my other options? Leading the discussion toward the patterns within the shapes, the students realize the strategy of an organized list is not efficient. They now switch strategies into drawing a picture, making it simpler, and looking for patterns. Of course, I reinforce the strategies of logical reasoning and brainstorming as vital elements to problem solving.

I allow the groups a few more minutes to complete the solution. Then the groups tell me how to solve the problem on the board. The actual solution is anticlimactic at this point because of the discussion that has lead up to it. The students then tell me what strategies they used from start to finish. One group did finish the problem by listing all the combinations; most groups used the pictures and patterns. The class discusses the validity and efficiency of both methods. At this point, the students have succeeded.

I now ask the students to think about a block of stamps 3×5 , or 4×5 , or 3×6 , etcetera. I ask if the solution for the 3×4 would apply to a different but similar problem. Because the students feel success, they willingly add their conjectures to the problem. The discussion for the extension problem again stresses the strategies rather than the answer. The goal of the problem solving exercise is to teach problem solving skills. The constant drilling of the strategies emphasizes the importance of the process rather than the solution.

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SELECTED PROBLEMS

1. At our school there are three academic clubs: Library Club, Science Club, and Math Club. Five students are members of all three clubs. One-third of the Library Club members and all the Math Club members belong to the Science Club. There are 24 members of the Library Club, 12 members of the Math Club, and 39 members of the Science Club. How many students belong only to the Science Club? (24 students)
2. Four men were shipwrecked on an island. The first day they gathered a pile of coconuts and then went to sleep. During the night one of the men woke up and ate one-third of the coconuts. A second man woke up and ate one-third of the remaining coconuts. A third man did the same. When the fourth man woke up, he ate one-fourth of the remaining coconuts. Then there were 6 coconuts left. How many coconuts did the men gather? (27 coconuts)
3. How many squares on a checker board? (204 squares)
4. During the recent census, a man told the census-taker that he had three children. When asked their ages, he replied, "The product of their ages is 72. The sum of their ages is the same as my house number." The census-taker ran to the door and looked at the house number. "I still can't tell," she complained. The man replied, "Oh, that's right. I forgot to tell you that the oldest one likes chocolate pudding." The census-taker promptly wrote down the ages of the three children. How old are they? (3,3,8)

5. Antarian, Polarian, and Vegan played a game with three cards. Each card had a different positive whole number on it. The rules were as follows:

1. Deal one card to each player.
2. Score the number of points indicated on your card.
3. Shuffle the cards and deal again to play the next round.

They played at least two rounds. After the last round, Polarian had 10 points, Antarian had 20 points and Vegan had 9 points. During the last round of play, Polarian scored 8 points. How many rounds did they play? How many points did each player score in each round? (They played 3 rounds.)

	A	P	V
Round 1	8	1	4
Round 2	8	1	4
Round 3	4	8	1
Totals	20	10	9

6. Twenty cards are chosen from a standard deck of playing cards – ten black cards, ace to ten, and ten red cards, ace to ten. The cards are arranged in a certain order so that, after completing the steps listed below, the cards turn up in order; black, ace to ten, then red, ace to ten.

Step 1: Hold the pack face down. Make two piles of cards by alternately placing one card up and one card down. Start with one card up.

Step 2: Pick up the cards that are face down. Repeat Step 1, placing the face-up cards on the first face-up pile.

Step 3: Repeat the process described in Steps 1 and 2 until all the cards are face up.

What is the original order of the twenty cards?

(1 black, 6 red, 2 black, 5 red, 3 black, 10 red, 4 black, 4 red, 5 black, 7 red, 6 black, 3 red, 7 black, 9 red, 8 black, 2 red, 9 black, 8 red, 10 black, 1 red)

7. From A to B there are four possible air routes. From B to C there are five possible air routes. From C to D there are three possible air routes. How many different trips can be taken from A to D and back without taking the same route on any section of the return trip? (1440)

8. Draw all the different ways you could buy four attached stamps. (19 ways)
9. A, B, and C decide to play a game of cards. They agree on the following procedure: When a player loses a game, he or she will double the amount of money that each of the other players already has. First, A loses a hand and doubles the amount of money that B and C each have. Then B loses a hand and doubles the amount of money that A and C each have. Then C loses a hand and doubles the amount of money that A and B each have. The three players then decide to quit, and they find that each player now has \$8. Who was the biggest loser? (A lost 5, B gained 1, C gained 4)
10. A figure composed of congruent cubic blocks has four layers. On the lowest layer are 7 rows of 7 blocks each. Centered on the bottom layer are 5 rows of 5 blocks each. Centered on top of that are 3 rows of 3 blocks each. Finally, a central block is placed on top of the entire structure. Then, the figure is painted, except for the bottom. How many blocks have painted faces?
- EXTENSION:
Suppose a similar figure has 99 rows of 99 blocks each on the bottom layer. How many blocks have painted faces?

# of faces	P	E
6	0	
5	1	1
4	0	0
3	12	196
2	36	9604
1	0	
0	35	

11. A rectangular block measuring 10 units by 8 units by 6 units is made up of cubes measuring 1 unit on a side. The base of the block is 10 units by 8 units. The outside of the block other than the base is painted red. How many of the unit cubes have exactly one face painted red? (188)
12. Three people each toss a penny at the same time. What is the probability that two people get the same side of the penny and the other person gets the opposite side? ($\frac{3}{4}$)

13. Given a 3-liter unmarked container, a 5-liter unmarked container, and an unlimited supply of water, can you obtain an accurate measure of 4 liters of water? You may pour from container to container or back into your water supply. (Yes, in 8 steps.)

3 liter	5 liter
0	0
3	0
0	3
3	3
1	5
1	0
0	1
3	1
0	4

14. Given a 4-liter unmarked container, a 7-liter unmarked container, and an unlimited supply of water, how can you obtain an accurate measure of 5 liters of water? What is the minimum number of pourings necessary? (8 pourings)

7 liter	4 liter
0	0
0	4
4	0
4	4
7	1
0	1
1	0
1	4
5	0

15. A cake in the form of a cube falls into a large vat of frosting and comes out frosted on all 6 faces. The cake is then cut into smaller cubes, each 1 inch on an edge. The cake is cut so that the number of pieces with frosting on 3 faces will be $\frac{1}{8}$ the number of pieces having no frosting at all. How many people will receive exactly 2 frosted faces? (48) Exactly 1 frosted face? (96) No frosted faces? (64) How large was the original cake? (6x6x6)

16. The counting numbers are arranged in five columns as shown below. Fill in the blanks to complete rows 5, 6, 7. In which column and row will the number 1000 appear?

	A	B	C	D	E
Row 1	1	2	3	4	5
Row 2	9	8	7	6	
Row 3		10	11	12	13
Row 4	17	16	15	14	
Row 5		18	19	20	21
Row 6	25	24	23	22	
Row 7		26	27	28	29
Row 250		1000			

17. I have a red die with 1, 2, 3, 9, 10, 11 on the faces. I have a green die with 3, 4, 5, 11, 12, 14 on the faces. Suppose you roll the two dice. How many different pairs of numbers can you possibly roll? (36)

When the two dice are rolled, the one showing the greater number is considered the winner. In how many cases would the red die win over the green die? In how many cases would the green die win over the red die? Would there be any ties? (Green - 25, Red - 9, Ties - 2)

G\R	1	2	3	9	10	11
3	G	G	T	R	R	R
4	G	G	G	R	R	R
5	G	G	G	R	R	R
11	G	G	G	G	G	T
12	G	G	G	G	G	G
14	G	G	G	G	G	G

18. Suppose that you and a friend are playing a game with two dice. One die is numbered 0, 1, 2, 3, 4, 5. The other die is numbered 1, 2, 3, 4, 5, 6. You and your friend take turns rolling both dice and scoring the number of points rolled. The winner of the game will be the player whose score is closest to 21. After two rolls of the dice one more time, what is the probability of maintaining or improving your score? (7/12)
19. One of two cubical dice has a blank face rather than one dot. The other die has a blank face rather than four dots. What is the probability that a sum of seven appears when the dice are thrown? (1/9)
20. Find a four-digit number so that if a decimal point is placed between the hundred's digit and the ten's digit, the resulting number is the average of the two-digit whole numbers on either side of the decimal point. (49.50)

Method to Your Mathness, 1992-1993 OPI "353" Teacher Training Grant
Selected References/Selected Problems for Problem-Solving

21. Dolores asked Norman to measure the edges of a rectangular box and to leave the information on her desk. On returning she found that Norman had left her the areas of the faces of the box. Knowing that the edges each measured a whole number of inches, she determined the length, width, and height of the box. If the areas of the faces were 120, 96, and 80 square inches, find the lengths of the edges of the box. What is the volume of the box?
Hint: area = length times width volume = length times width times height
($v = 8 \times 10 \times 12 = 960$ cubic inches)
22. A car travels at 80 kilometers per hour for 40 kilometers, 72 kilometers per hour for 48 kilometers, and then 96 kilometers per hour for 32 kilometers. What is the average rate of speed over the entire trip? (80 kilometers per hour)
23. In how many different ways can three people divide 25 pieces of candy so that each person gets at least 1 piece. (2300)
24. Sally is having a party. The first time the doorbell rings, one guest enters. If on each successive ring a group enters that has 2 more persons than the group that entered on the previous ring, how many guests will have arrived after the 20th ring? (400)
25. A hunter proudly said that he returned with 27 heads and 78 feet. If he brought back only ducks and rabbits, how many of each did he have? (Rabbits: 12 - Ducks: 15)
26. A survey of all the dogs and parakeets in a pet store shows that there are 15 heads and 42 feet. How many parakeets are there? (Parakeets: 9 - Dogs: 6)
27. There are several freight cars at a train station. The cars are either *long* or *extra-long*. The extra-long cars are twice the length of the long cars. (2 longs = 1 extra-long). Long and extra-long cars can be joined end-to-end to produce longer trains. How many *different* trains equal in length to 10 long cars can be formed from long and/or extra-long cars? (89)
28. Two bicycle riders, Jeff and Nancy, are 25 miles apart, riding toward each other at speeds of 15 mph and 10 mph, respectively. A fly starts from Jeff and flies toward Nancy and then back to Jeff again and so on. The fly continues flying back and forth at a constant rate of 40 mph, until the bicycle riders "collide" and crush the fly. How far has the fly traveled? (1 hour @ 40 mph = 40 miles)
29. The fifteen directors of the Round Tuit company always open their annual board meetings with a special ceremony in which each director shakes hands with each of the other directors. How many handshakes take place? (105)

30. A total of 28 handshakes was exchanged at a party. Each person shook hands exactly once with each of the others. How many people were at the party? (8)
31. An archeologist digging in a ruin found a set of measuring containers used by an ancient tribe of people. There were five different jars with the names MUG, LUG, PUG, BUG, and HUG on them. Use the following clues and place the correct name on each jar.
- a. A LUG is more than a BUG.
 - b. A MUG is never the least.
 - c. A LUG is not the greatest.
 - d. Only one thing is less than a HUG.
 - e. A PUG is more than a MUG.
 - f. More than one thing is greater than a MUG.
- (Small to large: PUG, LUG, MUG, HUG, BUG)
32. Merlin the Wizard discovered that two black bats would eat two pounds of fungus food every two weeks; three purple lizards would eat three pounds of fungus food every three weeks; & four variegated chameleons would eat four pounds of fungus food every four weeks. How many pounds of fungus food will 12 black bats, 12 purple lizards, & 12 variegated chameleons eat in 12 weeks? (Total: 156)
33. At one family reunion, every nephew was a cousin. Half of all uncles were cousins. Half of all cousins were nephews. There were 30 uncles and 20 nephews. No uncle was a nephew. How many cousins were neither nephews nor uncles? (5)
34. On planet Nuf, there are two kinds of Nuffians: lbs (which have 2 legs) and Etneps (which have 5 legs). When I walked into a room, I counted 39 Nuffian legs. How many lbs and how many Etneps were in the room?
(2 IBS, 7 ETNEPS; 7 IBS, 5 ETNEPS; 12 IBS, 3 ETNEPS; 17 IBS, 1 ETNEPS)
35. Maria has 4 math books, 3 novels, and 2 books of poetry. She wants to arrange them so that all the math books are together on the left end of the shelf, all the novels are together in the middle, and the books of poetry are together on the right end. How many different arrangements of the books are possible? (288)
What if 3 groups can be rearranged? (1728)
36. (A) A pail with 40 washers in it weighs 175 grams. The small pail with 20 washers in it weighs 95 grams. How much does the pail weigh alone? (15g) The washer alone? (4g)
(B) If a brick balances with three-quarters of a brick and three-quarters of a pound, then how much does the brick weigh? (3 lb.)

37. How many different 6-digit numbers can you make using the digits 1, 2, 5, 6, 7, and 9? (720) How many of these 6-digit numbers are divisible by 6? (240)
38. In the small town of Jollyville (population 6561) jokes travel fast. In one hour each person who hears a joke tells three other people who have not heard the joke, and then tells no one else. One morning a salesperson from out of town told the barber a new joke. How long did it take for everyone in Jollyville to hear the joke? (8 hours)
39. Farmer George has 100 yards of fencing. He wants to use it to make a plot for his garden. What should the dimensions of the garden be for:
- 1) the maximum area? (25x25)
 - 2) the minimum area? (1x49)
- Farmer Joe needs 100 square yards for his garden plot. What should the dimensions of the garden be for:
- 1) the minimum perimeter? (10x10)
 - 2) the maximum perimeter? (1x100)
40. The Talk-A-Lot Phone Company installs party lines with various numbers of customers. It charges \$7 a month per phone. After much research on costs, we discovered that installation and service of the phones will cost:
- | phones | cost |
|--------|------|
| 1 | \$2 |
| 2 | \$4 |
| 3 | \$7 |
| | etc. |
- If this pattern continues, how many phones can be put on a party line before the company would lose money? (12)
41. A pizza can be cut into seven pieces with three straight cuts. What is the largest number of pieces that can be made with eight straight cuts? (37)

42. Eight pills are on a table. All are the same size and color. However, one is poisonous and weighs a fraction more than the others. A balance scale may be used only twice to find the heavier pill. How can you be sure to find it?
Set 2 aside and balance 3 and 3. If they balance, balance the 2 you set aside. The heavy side is the poisonous pill. If they don't balance, take the 3 from the heavier side. Set 1 aside and balance other 2. If they balance, the one set aside is poisonous. If they don't balance, the heavier side is poisonous.
43. Colleen had some money. She gave \$20 to her brother for his birthday, and spent one-half of the remaining money for clothes. Then she bought gas for \$15 and used one-fifth of what was left for lunch. She saved the \$20 that she had left. How much money did she have to start with? (\$100)
44. Every day at noon a ship leaves San Francisco for Tokyo, and at the same instant a ship leaves Tokyo for San Francisco. Each trip lasts exactly eight days. How many Tokyo ships will each San Francisco ship meet? (17)
45. Samantha is experimenting with three spinners. Each spinner is divided into three equal sections with a number in each section. Spinner A shows 8, 6, 1; spinner B shows 7, 5, 3; spinner C shows 9, 4, 2. Samantha spins two spinners and records as winner the one that stopped on the greater number. She has already found that A usually wins over B and that B usually wins over C. She guesses that A will usually win over C. Is her guess correct? (Yes)
Find the probabilities:
A wins over B: $\frac{5}{9}$ - A wins
B wins over C : $\frac{5}{9}$ - B wins
A wins over C : $\frac{4}{9}$ - A wins
46. Suppose you have a three by four block of twelve postage stamps. You wish to give four stamps to a friend, all joined together with no stamp attached at the corner only. How many different ways could you give your friend these stamps? (65)
47. There is a train depot in each of the following cities in France: Paris, Marseilles, Fountainblue, Nice, Cannes, Versailles, Orleans, Bayonne, Lyon and LeMans. Each train depot must have a direct line of communication to each of the other depots.
How many direct lines are there between the depots in these ten cities? (45)
If a communications system is to be constructed among 100 cities, how many direct lines will be needed? (4,950)
How many direct lines will be needed for any number of cities? $\frac{n(n-1)}{2}$

48. At the last school board election in Angletown, 4620 votes were cast. Candidate Acute received 236 votes more than candidate Obtuse. Candidate Right received 698 votes more than candidate Acute. Candidate Straight received 256 votes less than candidate Right. How many votes did each receive?
(Acute = 929; Obtuse = 693; Right = 1627; Straight = 1371)
49. I am thinking of 2 two-digit numbers. They have the same digits, only reversed. The difference between the numbers is 54, while the sum of the digits of each number is 10. Find the 2 numbers. (82, 28)
50. In the game of UNDER, two dice are thrown. If the sum of the dots is under seven, the player receives 8 points. If the sum of the dots is seven or more, the player loses 6 points. How many points can a player expect to win or lose if 100 games are played? (16 losses)

MATH HINTS AND HELPS FOR ADULT LEARNERS

by
BILLINGS ADULT EDUCATION TEACHERS

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EXAMPLES AND EXPLANATIONS

Here are examples of short-cuts that have helped adults understand certain math concepts. When using these examples, it is important that the teacher write them out clearly and concisely so the student can use the operation later without the teacher.

WHOLE NUMBERS:

1. Addition--using tens:

The students only need to memorize the sums of tens and they will have a technique to use in all addition problems.

EXAMPLE:

Nine needs a one; eight needs a two; seven needs a three; six needs a four; five needs a five to make a TEN. When a student is adding $9+7$, the student knows 9 needs a 1 to make a 10. Therefore he takes 1 from the other number to make 10 and determines that the other number is now 6 ($7-1$). The answer is 16 ($10 + 6$).

2. Subtraction--borrowing with zeros

This method requires understanding of the values of numbers. The student needs to know the number that comes right before 20, 300, 4000, 100, etc.

EXAMPLE:

$$\begin{array}{r} 3,005 \\ - 636 \\ \hline \end{array}$$

$$\begin{array}{r} 299 \\ \underline{300} 5 \\ - 636 \\ \hline \end{array}$$

Because 6 cannot be subtracted from 5, the student needs to borrow. The student makes 5 into 15 and goes next door to borrow. Nothing is found in the tens place, so the student continues on to the hundreds place. Still there is nothing. The student continues on to the thousands place and finds the digit three. If the student has underlined as he goes to borrow, 300 is now underlined. The underlined number is the number the student is borrowing from. The student takes one from 300 and replaces it with 299. Now the student is ready to subtract.

EXAMPLE:

$$\begin{array}{r} 81,000 \\ - 62,345 \\ \hline \end{array}$$

$$\begin{array}{r} 099 \\ 8 \underline{100} 0 \\ - 62,345 \\ \hline \end{array}$$

In this example, the student needs to be more careful. After following the above procedure, 100 will be underlined. When the student subtracts 1 from 100, no (zero) hundreds and 99 remains. Make sure the student knows that the zero will go above the 1 and the two nines will go above the two zeros in 100. At this point, the subtraction will continue without much difficulty.

3. Multiplication--the sum of nines; double 2's to get 4's; double 4's to get 8's; double 3's to get 6's; forty-niners; patterns of facts

Once the student has learned the multiplication tables for twos and threes, these tricks can apply. Doubling the multiples of 2 will give the multiples of 4. Doubling the 4s gives the 8s. Doubling the multiples of 3 will give the multiples of 6. The only ones left to memorize are seven times itself and the multiples of nine. $7 \times 7 = 49$ ers, and the digits in the multiples of nine always add up to nine.

EXAMPLE:

$$4 \times 9 = ?$$

To help the student determine the first digit, remind the student that $4 \times 10 = 40$. Since 9 is not quite 10, $4 \times 9 =$ thirty-something. Or simply point out that the first digit in the answer will always be one less than the non-9 factor. Knowing the first digit is 3, the student can determine that $3 + 6 = 9$. Therefore 36 is 4×9 .

Multiplication Facts

Use this pattern to teach students the math facts on nines.

$$\begin{array}{rcl} \boxed{\quad 4 + \quad = 10} \\ 4 \quad \times \quad 9 \quad = 3 \quad 6 \\ \quad \quad \quad \text{tens} \quad \text{ones} \\ \boxed{4 - 1 = 3} \end{array}$$

$$\begin{array}{rcl} \boxed{\quad 7 + \quad = 10} \\ 7 \quad \times \quad 9 \quad = 6 \quad 3 \\ \quad \quad \quad \text{tens} \quad \text{ones} \\ \boxed{7 - 1 = 6} \end{array}$$

Another way of looking at the nines:

$$1 \times 9 = 9$$

$$2 \times 9 = 18$$

$$3 \times 9 = 27$$

$$4 \times 9 = 36$$

$$5 \times 9 = 45$$

$$6 \times 9 = 54$$

$$7 \times 9 = 63$$

$$8 \times 9 = 72$$

$$9 \times 9 = 81$$

Notice the digits in the products add up to

9. If a student is confused whether 9×6 is

54 or 56, have him add up the digits. If the sum is 9, then he has the correct product.

If students have a hard time with the multiplication facts of seven and eight, try to help them by getting them to remember certain word associations.

$$7 \times 7 = 49$$

Think of the football team.

$$8 \times 8 = 64$$

Think of the song. I find that if I sing the song to them they never forget what 8×8 is.

"Eight times Eight fell on the floor
picked it up 'twas sixty-four."

4. Division—using estimation; front into front; dividend is in the den; lining up
As the divisor gets larger, many students find long division gets more difficult.
Using estimation decreases this difficulty.

EXAMPLE:

When dividing 256 by 32, the student would determine that 32 is between 30 and 40. It is closer to 30 than it is to 40. (The student has just rounded the divisor to the nearest ten.) Using only the rounded tens place value, the student first takes 3 into the first digit in 256. Since 3 does not go into 2, the student then determines how many 3s in 25. 25 divided by 3 is 8. Next the student takes 8 times the actual divisor, 32, and gets 256. Therefore, 256 divided by 32 is exactly 8 with no remainder.

Remember: take the front into the front.

To help them identify the dividend, remind students "the dividend is in the den".

If a student is having a difficult time keeping the columns straight while doing long division, encourage the student to take lined paper and turn it on its side so that the lines are vertical instead of horizontal. Graph paper also works very well to assist the student with lining up the digits.

Keeping columns straight is especially important when the quotient of a division problem contains zeros in the middle of the number. In these problems it is important to inform students that they do not bring the number in the dividend down unless they are going to write their next number above it.

EXAMPLE:

$$\begin{array}{r} 2 \\ 27 \overline{) 567081} \\ \underline{54} \\ 27 \end{array}$$

Since the next number in the quotient will go here you may bring down the 7.

$$\begin{array}{r} 21 \\ 27 \overline{) 567081} \\ \underline{54} \\ 27 \\ \underline{27} \\ 0 \end{array}$$

Always stress to the student that they can bring the number in the dividend down only when they are going to write right above it.

DECIMALS:

1. Reading place values—reverse pyramid; place value helps

Show the student the reverse pyramid to help with reading place values:

1,000,000 .000001

100,000 .00001

10,000 .0001

1,000 .001

100 .01

10 .1

1

The numbers on the left side end in -s.

The numbers on the right side end in -ths.

2. Reading the value—change fractions to place value equivalents

Break the decimal down into its fractional components.

EXAMPLE:

$$\begin{array}{rclclclcl} .4567 = & \frac{4}{10} & + & \frac{5}{100} & + & \frac{6}{1000} & + & \frac{7}{10000} \\ & \frac{4000}{10000} & + & \frac{500}{10000} & + & \frac{60}{10000} & + & \frac{7}{10000} \end{array}$$

As the student reads all the numerators followed by the denominator, he has read the decimal number, 0.4567, correctly. It is read four thousand, five hundred sixty-seven ten-thousandths.

Students tend to have a hard time with place values to the right of the decimal point. Sometimes it helps to write out in words the different place values that are found to the right and left of the decimal point. After writing the following,

Hundreds Tens Ones . Tenths Hundredths

bring this to the attention of the student:

THERE IS ONLY *ONE* PLACE WHERE THE WORD *ONE* IS

Point out the continuing pattern of the place values, i.e., to the left of the decimal there is the 'ones-tens-hundreds' group, then the 'thousands' group, then 'millions', etc. Each group is separated by a comma. Within each group, the word one (understood), ten, and hundred is used, along with the group name, i.e., (one) thousand, ten-thousand, hundred-thousand, and so on for each new group. If students see this pattern, they realize they don't have to memorize so many different, unrelated words. The pattern to the right of the decimal point is the same, with the exception of no ones place, of course.

Students often count place value from either end of the number. Stress that the place values are counted from the decimal point in either direction.

One method to make sure the students get practice with place value is for you to read decimals as they should be read. .125 is not read "point one two five"; it is read "one hundred twenty-five thousandths."

3. Multiplication and division--powers of 10

Encourage the student to set up a multiplication problem like the ones below if one of the numbers ends in a zero or zeros. This helps keep the correct place value.

EXAMPLE:

$$\begin{array}{r} 245 \\ \times 900 \\ \hline 220500 \end{array}$$

$$\begin{array}{r} 9776 \\ \times 7000 \\ \hline 68432000 \end{array}$$

4. Comparing--write as fractions; add zeros

When comparing decimals, the student needs to write the decimals as fractions, then add zeros to the fractions to make equivalent denominators. Look at the numerators and determine which is larger.

EXAMPLE:

$$\begin{array}{l} .23 = \frac{23}{100} = \frac{230}{1000} \\ .023 = \frac{23}{1000} \end{array}$$

When using equivalent denominators, one need only compare the numerators to determine which is larger.

5. Fractions to decimals--fraction line means division

When changing fractions to decimals, remind the student that the fraction bar tells her to divide.

EXAMPLE:

$\frac{1}{2}$ Here the fraction bar tells the student to take 1 and divide it by 2 (or divide 2 into 1).

6. Decimals to fractions--3 R's

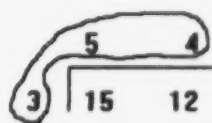
When changing decimals to fractions, remind the student about the 3 R's. Read it, (w)Rite it, and Reduce it.

FRACTIONS:

1. LCD—multiples; looking for factors not shared; dividing out shared factors; double division

I use a method for finding common denominators that I call "double division." It is actually Euclid's Algorithm put to good use. If I want to add the fractions

$\frac{8}{15}$ and $\frac{7}{12}$, I need to know the common denominator. This could cause a problem for many of our students. I put the 15 and the 12 both under the same division sign. Then I ask, "What goes into both numbers?" Then I perform the division.



Since 3, 5, and 4 have no common factor (other than 1), I multiply the $3 \times 5 \times 4$ to get the common denominator of 60. Another example: find the common denominator of 6, 8, and 9.



Notice that with 3 denominators, I deal with 2 at a time. Just carry the third number along. Keep dividing until there is no common factor except 1. Then multiply $2 \times 3 \times 1 \times 4 \times 3$ to get the common denominator of 72.

2. Reducing—knowing when done; with zeros; divisibility.
If the student can factor either the numerator or the denominator to its prime factors, and if none of these factors is common to the other fraction part, then the fraction is reduced as low as it goes.

EXAMPLE:

In the fraction $\frac{51}{34}$, the student would see 34 as an even number. Factoring 34 to its prime factors gives 2 and 17. The student then would check 51 and see if it contains either factor. 51 is not even so it does not have a factor of 2. The question remains, is 17 a factor? Dividing 17 into 51 gives an answer of 3. Therefore, this fraction will reduce, since both numerator and denominator share the common factor of 17. In reduced form it would be $\frac{3}{2}$.

Remind students that they can cancel one zero for one zero when reducing fractions. Note: they are actually dividing by 10 each time they cancel a zero.

EXAMPLE:

$$\frac{200}{600} = \frac{20}{60} = \frac{2}{6} = \frac{1}{3}$$

3. Subtraction--borrow one; make improper

When subtracting fractions, try using the following example when a student is struggling or misplacing the number that is being borrowed.

EXAMPLE:

$$\begin{array}{r} \overline{) 7 \frac{1}{3}} \\ \underline{5 \frac{4}{6}} \\ 2 \frac{1}{6} \end{array}$$

1 2/3

Another method of borrowing uses the "1" that students are familiar with:

EXAMPLE:

$$\begin{array}{r} 5 \frac{3}{4} = 5 \frac{6}{8} = 4 \frac{5}{1} \frac{6}{8} = 4 \frac{14}{8} \\ \underline{3 \frac{7}{8}} \\ 1 \frac{7}{8} \end{array}$$

4. Comparing--LCD; write as decimals; divisibility; treat top and bottom the same
To compare fractions, find their common denominator and change them to equivalent fractions all having that denominator. Then compare the numerators--the fraction with the largest numerator is the largest fraction.
Another way to compare fractions is to convert them to decimals and then compare them. (Refer to page 66, number 4.)

Rules of Divisibility

- 2: the last digit must be even (end in 0,2,4,6, or 8)
- 3: the sum of the digits must be divisible by 3
- 4: the last 2 digits must be divisible by 4
- 5: the last digits must be 0 or 5
- 6: must be divisible by both 2 and 3 (see rules above)
- 8: the last 3 digits must be divisible by 8
- 9: the sum of the digits must be divisible by 9
- 10: must end in 0

Stress "what you do to the top, you must do to the bottom."
Also "what you do to the bottom, you must do to the top."

PERCENTS:

1. Percent " T "; percent circle; percent triangle
Refer to section on the Percent " T " (pages 3 - 14).
2. Total—always 100%; part varies
Remind the student that the total always refers to 100% of something. The part can refer to something greater than or less than the total. Many students expect the larger number to be the total and the smaller one to be the part. It is important to remind them that this is not always true.
3. Conversions— $1/2 = .5 = 50\%$
To help memorize the 6 conversion rules, $1/2 = .5 = 50\%$ is helpful. This example can be used to build the pattern for the conversions and is easy to memorize.
4. Percents to decimals—L
When changing percents to decimals, the student often is confused on whether to move the decimal two places to the right or the left. Let the L in decimalL remind the student to move the decimal to the Left to get a decimalL.
5. Decimals to percents—R
Let the R in peRcents remind the student to move the decimal to the Right to get a peRcent.
6. Another way to remember which direction to move the decimal point when changing from decimals to percents is this: Picture the alphabet—D is to the left of P. Therefore, if I am going from D (decimal) to P (percent), I go to the right. If I am going from P (percent) to D (decimal) I must go to the left.

SCIENTIFIC NOTATION (TENS):

1. Positive, zero and negative powers of 10

Many students have a difficult time with zero and negative exponents. Showing the effect of the decreasing exponent helps them grasp this concept.

EXAMPLE:

$$\begin{array}{rclcl} 1000 & = & 10^3 \\ 100 & = & 10^2 \\ 10 & = & 10^1 \\ 1 & = & 10^0 \\ \frac{1}{10} = .1 & = & 10^{-1} & = & \frac{1}{10^1} \\ \frac{1}{100} = .01 & = & 10^{-2} & = & \frac{1}{10^2} \\ \frac{1}{1000} = .001 & = & 10^{-3} & = & \frac{1}{10^3} \end{array}$$

2. Multiplying—commute the multiples of 10

To help the student grasp a quick way of multiplying with powers of tens, use the commutative law to arrange the powers together.

EXAMPLE:

$$\begin{aligned} 34,000 \times 2,000,000 &= 3.4 \times 10^4 \times 2 \times 10^6 \\ &= 3.4 \times 2 \times 10^4 \times 10^6 \\ &= 6.8 \times 10^{10} \end{aligned}$$

GEOMETRY:

1. History of pi-wheel

Once upon a time there was a wheel maker who was trying to wrap his wooden wheel. He found that a 1 yard diameter wheel needed a little more than 3 yards to wrap it. A 2 foot diameter wheel needed a little more than 6 feet to wrap it. This was his understanding of pi.

2. Visualization of measures

When explaining measurements to adults, the teacher should use manipulatives or pictures with the students. When talking about areas, use examples that are relevant to adults, i.e., laying carpet, painting walls, putting weed-and-feed on the lawn, or putting up wallpaper. Some good examples for perimeters would be fencing, molding, and wallpaper borders. Volume examples would include filling a flowerbed with decorative rock or bark, or pouring a concrete sidewalk or driveway. Since adult students have so many real life experiences, draw upon their backgrounds (shopping, baking, painting, carpentry, household repair, etc.).

3. Shortcut unit conversion

A shortcut method to dimensional analysis works very well in conversion problems.

EXAMPLE:

A street 20 feet by 150 feet needs to be paved. The asphalt will be spread 6 inches deep and costs \$36 per square yard. How much will the paving cost?

Since it is necessary to convert 20 feet and 150 feet to yards, they will each be divided by 3. The 6 inches will have to be divided by 36. The setup, which is done all at once, will look like this:

$$\frac{20}{3} \cdot \frac{150}{3} \cdot \frac{6}{36} \cdot \frac{36}{1}$$

ALGEBRA:

1. Substitution—use $V = LWH$

Students can often relate to a substitution for L, W, and H in the volume formula. Knowing this as a substitution helps the student identify with substitution in an algebraic equation.

EXAMPLE:

Find V when $L = 4$; $W = 3$; $H = 2$.

Using the formula $V = LWH$, the student substitutes 4 for L; 3 for W; and 2 for H.

This results in the following new equation: $V = 4 \times 3 \times 2$.

2. Solving—cancel operations on variable; scale of equality

When solving equations, just think about canceling whatever operation is being done to the variable.

Solving for an unknown can be a difficult concept for an adult student. This is especially true if the student has not had algebra previously. A good visual representation for the equal sign is a scale: whatever they do on one side of the equal sign they must do on the other side to balance out the scale.

Stress "what you do to one side, do to the other"

"When you switch sides, you switch signs." It's enough of a tongue twister that the students remember it.

3. Terminology—natural, whole, integer, rational, irrational

Natural Numbers—Numbers we naturally count. We never count something unless there is at least one of them.

Whole Numbers—Includes the HOLE. Doesn't zero look like a hole?

Integers—Just the negatives of the natural numbers included with those above.

Rational Numbers—Numbers that can be written as ratios (fractions). Remember that all the integers can be written as fractions by putting them over 1. Rational numbers include all decimals that either terminate or repeat.

Irrational Numbers—Numbers that can never be written as ratios (fractions). This includes all decimals that neither terminate nor repeat. Remind the student that pi is irrational and the value for pi that is used in math is only a close approximation of true pi.

4. Writing—expressions & equations—use 10 or 1

When the student is learning to write word phrases as algebraic expressions, they have difficulty with phrases like:

→ less than

→ diminished by

→ decreased by

If the student replaces the unknown number in the problem with 10 or 1, he will find a pattern to help him decide what is going to be subtracted from the other.

5. Factoring polynomial coefficients—start with 1 until they meet
Looking for all the factors in the polynomial coefficients is sometimes difficult for the student. If the student starts with 1 and continues to check each consecutive number until reaching the corresponding factor, there will never be a factor missing.

EXAMPLE:

$$18x^2 + 57x + 24$$

Before factoring this polynomial, the student needs to find all the factors of the coefficients 18 and 24.

$$18 = 1 \times 18$$

$$2 \times 9$$

$$3 \times 6$$

4 does not go into it evenly.

5 does not go into it evenly.

6 already has been used as a factor. All the factors have been found.

The factors are 1, 18, 2, 9, 3, and 6.

Now look at the factors of 24.

$$24 = 1 \times 24$$

$$2 \times 12$$

$$3 \times 8$$

$$4 \times 6$$

5 does not go into it evenly.

6 already has been used as a factor. All the factors have been found.

The factors are 1, 24, 2, 12, 3, 8, 4, and 6.

6. Variables—numbers that vary
To help students with the term variable, tell them it is the number that varies. Instead of writing all the numbers it could be, it is written as a letter and represents all those numbers it could be at different times.
7. Addition
As soon as the student is developmentally ready, switch him or her from using two signs (adding a negative) to using one sign.
8. Subtraction—1 term—negative; more terms—change signs
Algebra books often teach subtraction as "change the sign and add". When only one term is being subtracted, it is easier to think of it as a negative number rather than a number being subtracted. To work with the negative number that is subtracted, think of the double-negative we use in English. "I do not think that you're not going to win." That is the same as saying that I think you are going to win. The double negative makes a positive in math also. Make sure when presenting this idea to stress that the double negative must be on one term.

EXAMPLE:

-5 - 7, is the same as negative 5 and negative 7, which is negative 12.

-5 - (-7), is the same as negative 5 and double negative (one big positive) 7, which is positive 2.

9. Multiplication

If a student is having a hard time with the multiplication of integers, remind her or him that two like signs result in a positive answer and two unlike signs give a negative answer.

A cute story that helps the student remember the multiplication rules: let + stand for good and - stand for bad.

(+ + = +) If a good thing happens to a good person, that's good.

(+ - = -) If a good thing happens to a bad person, that's bad.

(- + = -) If a bad thing happens to a good person, that's bad.

(- - = +) If a bad thing happens to a bad person, that's good.

10. Exponents—with & without parentheses

Students often need to be reminded about the function of exponents when parentheses are or are not present. The exponent only relates to the variable to the left of the exponent, unless grouping with parentheses includes more variables.

EXAMPLE:

$$xy^2 = xyy$$

$$(xy)^2 = x^2 y^2 = xxyy$$

$$-2^2 = -(2)(2) = -4$$

$$(-2)^2 = (-2)(-2) = 4$$

The negative sign is separate from the digit 2. The exponent does not relate to the negative unless the negative is grouped with the 2.

11. Order of operations: PEMDAS: Please Excuse My Dear Aunt Sally. This is an easy way to remember Parentheses, Exponents, Multiplication & Division, Addition & Subtraction.

MISCELLANEOUS:

1. Inequalities— $>$ and $<$

A different way to think about an inequality is to see it as having two opposite ends: the big end and the little end. Remind the student that the large number is on the big end side, and the smaller number is on the little end side.

"Pac-man eats the biggest one."

2. Implied facts—signs $+$; coefficients, denominators, and exponents 1; roots 2; decimals at the right

Students should know that there are some mathematical facts that are implied but usually not written:

If no sign is on a number, it is $+$.

If a variable has no coefficient shown, it is 1.

If a number has no denominator shown, it is 1.

If a number has no exponent shown, it is 1.

If a radical has no root shown, it is a square root.

If a number has no decimal shown, it is at the right of the number (a whole number).

3. Prime & composite—2 factors or >2

A prime number has only 2 factors, 1 and itself.

Composite numbers have two or more factors.

4. Ratios—use of

When explaining ratios to adult students, give them examples of how ratios are used in the world around them. Ratios can be used for doubling recipes or cutting them back by $1/3$. If a student is interested in sewing, explain how different size patterns can be made by using ratios. Ratios are also used in woodworking.

Another common use of ratios is in mixing things like liquid fertilizer, insect spray, or gas and oil for a lawn mower, chain saw, or weedeater.